

**I-** A thin plate covers the triangular region bounded by  $x$ -axis and the lines  $x = 1$  and  $y = 2x$  in the first quadrant. The plate's density at point  $(x,y)$  is:  $\delta(x; y) = 6x + 6y + 6$ . (5-pts)

a. Draw the specified region of integration.

b. Find the plate's:

i. Mass.

ii. First moments.

a. Deduce the center of mass of the given plate.

**II-** Evaluate the integrals:

a.  $\int_0^1 \int_0^{x^2} 3x^3 e^{xy} dy dx.$

b.  $\int_0^3 \int_0^2 (4 - x^2) dy dx.$

**III-** Prove that the value of the iterated integral:

$$\int_0^2 \int_0^{x+y} \int_0^x e^x (y + 2z) dz dy dx = 19\left(\frac{e^2}{3} + 1\right).$$

**IV-** Consider the following region  $R$ : (6-pts)

a. Calculate over the given region:

i.  $M = \iint_R 1 dy dx.$

ii.  $M_1 = \iint_R x dy dx.$

iii.  $M_2 = \iint_R y dy dx.$

b. Deduce the coordinates of the point  $G\left(\frac{M_1}{M}, \frac{M_2}{M}\right)$

