

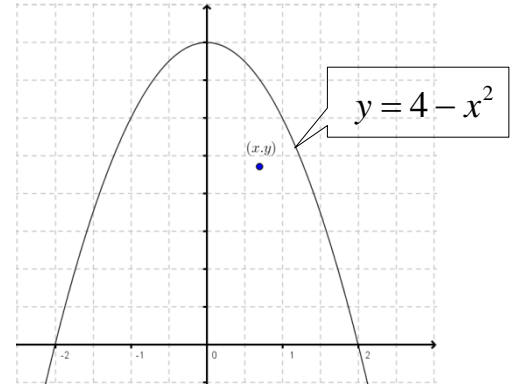
Answer *three* out of the four given questions:

- I-** Consider the region  $R$  bounded by the curves  $y = x$  and  $y = x^2$  (7-pts)
- Draw the above region  $R$ .
  - Find the area of the region  $R$ .
  - Prove that the centroid of the given region  $R$  is of coordinates  $(\frac{1}{2}, \frac{2}{5})$ .

- II-** Consider the parabolic region  $R: 0 \leq y \leq 4 - x^2$ .

(See adjacent figure).(7-pts)

- Recopy the given region and indicate the bounding curves.
- Use double integration to evaluate the mass of the lamina:  $M$  of density function  $\delta(x, y) = 2x$
- Use the above part to find the center of mass of the given lamina is of coordinates  $(0; \frac{4}{3})$ .



- III-** Sketch then calculate the integral  $I$  bounded by a rectangular region: (6-pts)

$$I = \int_0^3 \int_0^2 (4 - x^2) dy dx.$$

- IV-** A solid right (non-circular) cylinder has its base  $R$  in the  $xy$ -plane and is bounded from above by the paraboloid:  $z = x^2 + y^2$ . (6-pts)

- Sketch the region  $R$  in the  $xy$ -plane: which is enclosed by the lines  $y = x$ ,  $x = 0$  &  $x + y = 2$ .
- Express the volume of the region that lies under the paraboloid and above the region  $R$ , by a double integral.
- Prove that the volume of this region is:  $V = \frac{4}{3} \text{units}^3$ . (Note:  $(a - b)^3 = a^3 - 3a^2b + 2ab^2 - b^3$ )

*Good Work.*