

**I-** Integral in rectangular regions: (4-pts)

Consider the double integral:  $I = \int_0^3 \int_{-2}^0 (x^2 y - 2xy) dy dx$ .

1. Determine the limits of  $x$  &  $y$ .
2. Draw the specified region of integration.
3. Prove that  $I = -\frac{8}{3}$

**II-** Double integrals as volume: (4-pts)

Consider the region  $R$  enclosed between the straight lines  $(d): y = x$ ,  $(\Delta): x = 0$  and  $(l): y = -x + 2$ .

1. Determine the coordinates of  $A$ , the point of intersection of the straight lines  $(d)$  &  $(l)$ .
2. Sketch the region  $R$  and plot the point  $A$ .
3. Consider the surface  $z = f(x, y) = x^2 + y^2$ .
  - a. Specify the limits of integration of the integral of:  $V = \iint_R f(x, y) dy dx$
  - b. Prove that:  $V = \frac{448}{3}$
  - c. What does the value  $V$  of represent?

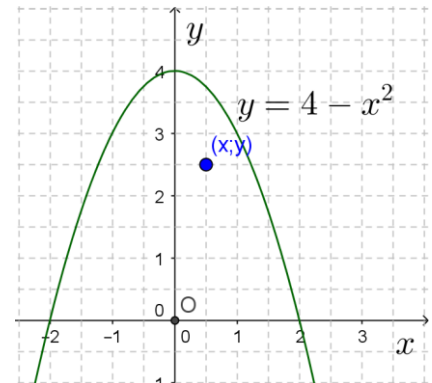
**III-** Consider the region  $R$  bounded by the curves  $y = x$  and  $y = x^2$  (6-pts)

- a. Draw the above region  $R$ .
- b. Find the area of the region  $R$ .
- c. Prove that the centroid of the given region  $R$  is of coordinates  $(\frac{1}{2}, \frac{2}{5})$ .

**IV-** Consider the parabolic region  $R: 0 \leq y \leq 4 - x^2$ .

(See adjacent figure).(6-pts)

- 1) Recopy the given region and indicate the bounding curves.
- 2) Use double integration to evaluates the mass of the lamina:  $M$  of density function  $\delta(x, y) = 2x$
- 3) Use the above part to find the center of mass of the given lamina is of coordinates  $(0; \frac{4}{3})$ .



Good work