التجربة الأولى لعام 2007 - 2008

الشهادة المتوسطة

مسابقة في الرياضيات الانكليزي المدّة: ساعتين الإسم: الرقم:

ارشادات عامة: – يسمح باستخدام آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البينات.

يستطيع المرشح الاجابة بالترتيب الذي يناسبه دون التزام بترتيب المسائل الواردة في المسابقة.

عدد المسائل 6 وجميعها إلزامية.

العلامة القصوى 20.

## 1st exercise: (3pts)

Choose the correct answer and justify

No.	Questions	Answers		
		a	Ъ	С
1.	the equation $\frac{4x^2 - 9}{2x + 3} = 0$	has a unique solution $x = \frac{-3}{2}$	has 2 solutions $x = \frac{-3}{2} \text{ and } x = \frac{3}{2}$	has a unique solution $x = \frac{3}{2}$
2.	Given a triangle ABC of sides $AB = \sqrt{2 + \sqrt{3}} cm$ , $AC = \frac{\sqrt{6} + \sqrt{2}}{2}$ and $BC = 1 + \sqrt{3} cm$ then	triangle ABC is isosceles at B	triangle ABC is scalene	triangle ABC is right and isosceles at A
3.	$\frac{4^{152} - 2^{303}}{8^{101}} =$	2	1	3
4.	The two circles $C(0,\sqrt{8})$ and $C'(0',\sqrt{18})$ such that $00' = \sqrt{50}$ are	internally tangent	not intersecting	externally tangent

## 2<sup>nd</sup> exercise: (3pts)

(C) is a circle of center O and diameter [AC] such that  $AC = 5\sqrt{10} - 2\sqrt{40} + \sqrt{2} \times 3\sqrt{5}$ . B is a point such that  $BO = (\sqrt{5} + \sqrt{2})^2 + (1 - 2\sqrt{2})(1 + 2\sqrt{2})$  (unit of length is dm)

- 1. Write AC in the form  $a\sqrt{b}$  where a and b are two natural integers to be determined. (34 pt)
- 2. Show that  $BO = 2\sqrt{10}$  and justify that B belongs to (C) (1 \(\frac{1}{4}\) pts)
- 3. Calculate AB if  $BC = 4\sqrt{5}$  then deduce that the triangle ABC is right isosceles at B(1pt) Remark: you can only draw a sketch

#### 3<sup>rd</sup> exercise: (2 ½ pts)

Consider the two polynomials:  $P(x) = (a^2 - 5)x^2 + 6x + c - 2$  and  $Q(x) = -4x^2 + (b^2 - 10)x - 2$ 

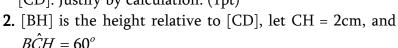
- 1. Find a, b, and c so that P(x) and Q(x) are identical. (1pt)
- 2. Find b so that x = 1 is a root of Q(x). ( $\frac{1}{2}$  pt)
- 3. a) Verify that  $-4(x-1)(x-\frac{1}{2}) = -4x^2 + 6x 2$  (½ pt)
  - b) Find the roots of Q(x) when  $b = 4 (\frac{1}{2} pt)$

# 4th exercise: (5 ½ pts)

**Part A:** Consider the two real numbers:  $x = \frac{36}{\sqrt{3}} - 2\sqrt{27} + 2\sqrt{4}$  and  $y = 4\sqrt{75} - \frac{3\sqrt{96}}{\sqrt{2}} + 2\sqrt{4}$ 

- 1. Show that  $x = 4 + 6\sqrt{3}$  (  $\frac{3}{4}$  pt)
- 2. Write y in the form of  $a + b\sqrt{3}$  where a and b are two integers to be determined. (34 pt)

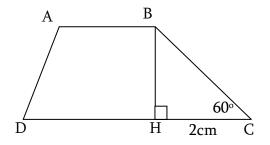
**Part B: 1**. Let x and y represent the lengths of the bases of the trapezoid ABCD. Indicate the length of [AB] and that of [CD]. Justify by calculation. (1pt)



a. Show that BC = 4cm, then deduce BH using two different ways. (1  $\frac{1}{2}$  pts)

b. Give an approximate value rounded to the nearest 0.001 of BH. (  $\frac{1}{2}$  pt)

c. Calculate the exact area of the trapezoid ABCD (1pt)



## 5<sup>th</sup> exercise: (3pts)

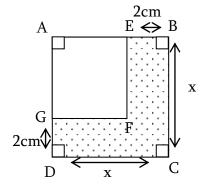
Given a square ABCD of side x cm (x > 2).

The sides [AB] and [AD] are decreased by 2cm each to obtain AEFG.

1. Find the nature of AEFG (1pt)

2. Determine the area of the shaded region in the form of a(x - b) where a and b are natural integers to be determined. (1pt)

3. Deduce the value of x so that the area of the shaded region is  $16cm^2$ . (1pt)



### 6th exercise: (3pts)

Given a circle (C) of center O and radius R. [AB] is a fixed diameter and M is a variable point on (C). The circle ( $C_1$ ) of center M and radius MA cuts (C) in A and E. Let D be the point diametrically opposite to A in ( $C_1$ ).

1. Draw a figure. (  $\frac{1}{2}$  pt)

2. Determine the nature of each of the following triangles ADE and AEB. Deduce that the points B, E and D are collinear. (1pt)

3. Show that (OM) and (BD) are parallel. Deduce that BD = 2R (  $\frac{3}{4}$  pt)

4. Choose the correct answer and justify.

a. The point B is

i. variable

ii. fixed

b. The point D is

i. variable

ii. fixed

c. The length BD is

i. constant

ii. variable

From the above questions, deduce the path on which the point D moves when M moves on (C). ( 3/4 pt)