مسابقة في الرياضيات الانكليزي الريّة : ساعتين : الرقم :

إرشادات عامة:
-- يمكن الإجابة على ألسائلّ بالترنيب الالذي تريد - يرجى الإجابة بخط واضح ومرتب

- العلامة القصوى من 30


## $1^{\text {st }}$ exercise: (5pts)

For each question, indicate the correct answer, and then justify.


## $\underline{2}^{\text {nd }}$ exercise: (6pts)

1) Given that $S=\frac{1}{a}+\frac{1}{2 a}+\frac{1}{3 a}+\frac{1}{6 a}$; where $a$ is a non-zero integer.
a. Write $S$ in the form of a fraction. ( pt)
b. Deduce that we can write the fraction $\left(\frac{2}{7}\right)$ as the sum of four fractions.(1pt)
2) Given the rectangle $A B C D$ such that: $A B=\sqrt{4+\sqrt{7}}$ and $B C=\sqrt{\frac{7}{2}}+\sqrt{\frac{1}{2}}$.
1. Calculate $A B^{2}$ and $B C^{2}$. Deduce that $\mathrm{AB}=\mathrm{BC} .\left(1^{11 / 2} \mathrm{pts}\right)$
2. What can you say about the rectangle $A B C D$ ?(pt)
3. Develop and reduce $(\sqrt{7}+1)^{2} \cdot(1 / 2 \mathrm{pt})$
4. Calculate the radius of the circle circumscribed about triangle ABC. (pt)

## $3^{\text {rd }}$ exercise: (5pts)

1. Consider the polynomial: $\mathrm{p}(\mathrm{x})=3 \mathrm{x}^{3}-12 \mathrm{x}^{2}-3 \mathrm{x}+\mathrm{a}$. Calculate a so that 1 is a root of $\mathrm{P}(\mathrm{x}) \cdot(1 / 2 \mathrm{pt})$
2. In what follows suppose that $\mathrm{a}=12$.

Factorize $P(x)$, and then solve the equation $P(x)=0 .\left(1 \frac{1}{2} p t s\right)$
3. If $\mathrm{Q}(\mathrm{x})=(\mathrm{x}-7)\left(\mathrm{x}^{2}+2 \mathrm{x}+1\right)+(\mathrm{x}-1)(\mathrm{x}+1)^{2}$. Prove that the given polynomial can be written in the form $Q(x)=(2 x-8)(x+1)^{2} .(1 p t)$
4. Let $\mathrm{E}(\mathrm{x})=\frac{\mathrm{P}(\mathrm{x})}{\mathrm{Q}(\mathrm{x})}$
a. Find all values of $x$ for which $E(x)$ is defined and then simplify $E(x) .(1 p t)$
b. Find $E(\sqrt{2})$, then rationalize the denominator of the answer obtained. (1pt)

## $4^{\text {th }}$ exercise: ( 3 pts )

In the opposite figure $A B C D$ is a rectangle and $D E F G$ is a square.
a. Express the areas of $A B C D$ and DEFG as a function of $x$. ( 1 pt )
b. Determine the value of $x$ if the area of the shaded region is $26 \mathrm{~cm}^{2}$. $(1 \mathrm{pt})$
c. Find the dimensions of $\mathrm{ABCD} .(1 \mathrm{pt})$

## $5^{\text {th }}$ exercise: (5pts)



Let (C) be a circle of center $O$, radius $r=5 \mathrm{~cm}$ and diameter [ AB ]. The perpendicular bisector of [AO] intersect (C) at points D \& E. $M$ is the symmetric of O with respect to A .

## 1. a. Prove that:

ii. The measure of angle $\mathrm{D} \hat{A} O$ is $60^{\circ} .(3 / 4 \mathrm{pt})$
iii. The triangle MDO is right at $D$. $(3 / 4-\mathrm{pt})$
b. What do the lines (MD) and (ME) represent with respect to the circle (C).Justify. (1pt)
c. Calculate the measure of the angles DME and $A \hat{D} B .(1 \mathrm{pt})$
d. Calculate the length of the segments [MD] and [ME]. (1pt)
2. Complete the following sentence: ( $1 / 2 \mathrm{pt}$ )

From the point M outside the circle, we can draw $\qquad$ so that MD $\qquad$ ME.

## $6^{\text {th }}$ exercise: (6pts)

Let $A B C$ be a right triangle at $A$, and $[A H]$ the height issued from $A$. See opposite figure.
The point Eis the symmetric of $H$ with respect to the line ( AB ), and $F$ is the symmetric of H with respect to the line (AC).

1. Reproduce this figure. ( $1 / 2 \mathrm{pt}$ )
2. a. Prove that $A$ is the center of the circle (C) circumscribed about the triangle $E H F .(1 \mathrm{pt})$
b. Deduce that the points E, A, and F are collinear. (1pt)

3. In this part, take $A B=6 \mathrm{~cm} \& A C=2 \mathrm{~cm}$.
a. Show that $B C=2 \sqrt{10} \mathrm{~cm} .(1 \mathrm{pt})$
b. Calculate the area of triangle ABC , and then deduce that $\mathrm{AH}=\frac{3 \sqrt{10}}{5} \mathrm{~cm} .\left(1 \frac{1}{2} \mathrm{pts}\right)$
4. Answer by true or false and justify your answer: Is (BC) tangent to the circle (C) at point H ? (1pt)
