التجربة الأولى لعام 2009 - 2010 | الشهادة المتوسطة

مسابقة في الرياضيات الانكليزي	المدّة : ساعتين	الإسم :	الرقم :

إرشادات عامة:

- يسمح بإستعمال ألة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على ألمسائل بالترتيب االذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 30
- عدد المسائل: 6

1st exercise: (4pts)

Decide if each statement is true or false. Justify.

1. Given a triangle ABC such that :
   $AB = \left(\frac{3}{4} \times \frac{4}{5} \div \frac{4}{5}\right) \times \left(3.5 - \frac{3}{2}\right)$ and $BC = 0.015 \times 10^2$; then triangle ABC is isosceles at B. (1pt)
2. Given a rectangle ABCD such that $AB = \sqrt{4 + \sqrt{7}}$ and $BC = \sqrt{\frac{7}{2}} + \sqrt{\frac{1}{2}}$, then ABCD is a square. (1pt)
3. $\sqrt{24}$ is half of $\sqrt{48}$. (½ pt)
4. $\sqrt{\left(2 - \sqrt{5}\right)^2} + \frac{4\sqrt{5} - 5}{\sqrt{5}}$ is a natural integer. (1pt)
5. The expanded form of $\left(-\frac{3}{2}x - 4\sqrt{5}\right)^2$ is $\frac{9}{4}x^2 + 80$. (½ pt)

2nd exercise: (5pts)

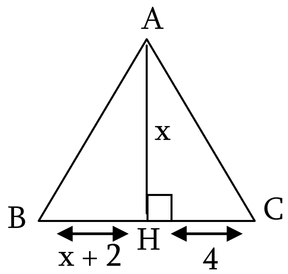
Part A:

Consider the two polynomials $P(x) = x^2 + 6x - 16$ and $Q(x) = (x + a)^2 - 25$.

1. Determine the value of a so that P(x) and Q(x) are identical polynomials. (1pt)
2. Suppose in this part that a = 3. Deduce the roots of P(x). Interpret the meaning of the roots obtained.(1½pts)

Part B:

Consider a triangle ABC (see figure to the right).



1. Show that the area A(x) of triangle ABC is written as $A(x) = \frac{x^2 + 6x}{2}$. (1pt)
2. Calculate x if A(x) = 8. Are the values of x thus obtained accepted or rejected? Justify. (1 ½ pts)

3rd exercise: (4½pts)

Consider the following number A:

$$A = \sqrt{23.36\overline{1}} + \frac{7 \times \sqrt{3^4 \times (108)^3}}{\sqrt{75 \times 3^8 \times 6^4}}$$

1. Show that $\sqrt{23.36\overline{1}} = \frac{29}{6}$. (1pt)
2. Verify that $\frac{7 \times \sqrt{3^4 \times (108)^3}}{\sqrt{75 \times 3^8 \times 6^4}} = \frac{14}{5}$. (1 ½ pts)
3. Calculate A, then write A^{-2} in scientific notation. (2pts)

4th exercise: (5pts)

Consider the algebraic expression: $A=(x-\sqrt{2})^2-8$

1. Develop, reduce, and order A in the ascending order of the powers of x. (½ pt)
2. Calculate A when $x=\sqrt{18}$. (½ pt)
3. Show that a factorised form of A is: $(x-3\sqrt{2})(x+\sqrt{2})$. (¾ pt)
4. Given $B=x(x-3)+x\sqrt{2}-3\sqrt{2}$. Write B as a product of two factors of first degree each. (¾ pt)
5. Consider $F=\frac{A}{(x-3)(x+\sqrt{2})}$
   a) For which values of x, the fractional expression F is defined? (½ pt)
   b) Simplify F. (½ pt)
   c) Calculate the numerical value of F when $x=2\sqrt{2}$ and give the answer as a rational number in the denominator. (1 ½ pts)

5th exercise: (6½pts)

Consider a semi – circle (C) of center O and diameter AB = 12cm. H is the midpoint of [AO] and (d) is the perpendicular to (AB) at H that cuts the semi – circle (C) at M.

1. Draw a figure .(½ pt)
2. What is the nature of triangle AMO? Deduce the length AM then MH. (1 ½ pts)
3. Determine the nature of triangle AMB. Deduce the exact value of MB, then give an approximate value to the nearest 10^{-3}. (2pts)
4. The perpendicular bisector of [AB] cuts (AM) at E.
   a) Show that the triangle ABE is equilateral. (½ pt)
   b) Show that M is the midpoint of [AE]. (1 pt)
   c) Calculate OE then deduce that OE = MB. (1 pt)

6th exercise: (5pts)

Consider a rectangle ABCD such that BD = 2BC and let O be its center. (see figure below).

1. Reproduce the figure. (¾ pt)
2. The perpendicular bisector of [BD] intersects [AB] at M and [CD] at N.
   a) Show that the two triangles MBO and NDO are congruent. Deduce that BM = DN. (1pt)
   b) Show that BMDN is a rhombus. (1 ¼ pts)
3. Show that the triangle OBC is equilateral. Deduce the measure of $M\hat{B}O$. (1pt)
4. Show that [BN) is the bisector of $O\hat{B}C$. (1pt)