ليسه دي زار

الشهادة المتوسطة — التجربة الأولى لعام 2010 - 2011

مسابقة في الرياضيات الانكليزي	المدّة : ساعتين	الإسم :	الرقم :

إرشادات عامة:
- يسمح بإستعمال ألة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على ألمسائل بالترتيب الذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 30
- عدد المسائل: 5

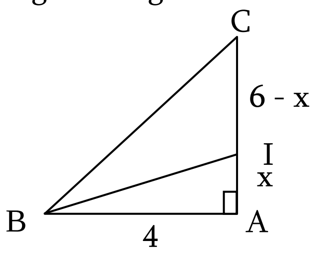
1st exercise: (3pts)

Given the real numbers: $A=\frac{7}{18}\times\frac{2}{7}-\left(\frac{5}{3}-1\right)^2$, $B=\frac{3\times10^2\times5\times10^4}{12\times\left(10^3\right)^3}$, $C=\sqrt{250}-\sqrt{490}+2\sqrt{81}$

1) Write A in the form of an irreducible fraction. (¾ pt)
2) Verify that B can be written in the form of $2^m\times5^n$ where m and n are integers to be determined, then verify that B is a decimal number. (1pt)
3) Simplify C and determine its inverse without a radical in the denominator. (1 ¼ pts)

2nd exercise: (6pts)

In the table below, only one among the proposed answers to each question is correct. Write down the number of each question and give, **with justification**, its correct answer.

No.	Questions	Answers		
		a	b	c
1.	$a=\left(4\sqrt{5}-2\right)\left(\sqrt{5}+1\right)-\left(\sqrt{5}-1\right)^2$ is	an integer	a positive irrational number	a negative irrational number
2.	Among $\sqrt{7.\overline{1}}$; $\sqrt{18}$; $\sqrt{8^2-4^2}$; $\frac{\sqrt{48}}{\sqrt{12}}$; $\sqrt{10^{10}}$	one irrational number	two irrational numbers	three irrational numbers
3.	If **x** is a negative real number then $\sqrt{\frac{x^2}{4}+\frac{4x^2}{9}}=$	$\frac{5x}{6}$	$\frac{-5x}{6}$	does not exist
4.	Given a right triangle ABC at A such that  If triangle IBC is isosceles at I then x =	$\frac{3}{5}$	$\frac{5}{3}$	$\frac{13}{3}$
5.	Calculate the numerical value of : $4a^2+\frac{25}{a^2}$ if $2a+\frac{5}{a}=8$	can't be calculated	64	44
6.	$-5^2-3^2-7^2=$	$\left(-15\right)^2$	- 83	-15^2

3rd exercise: (7 ½ pts)

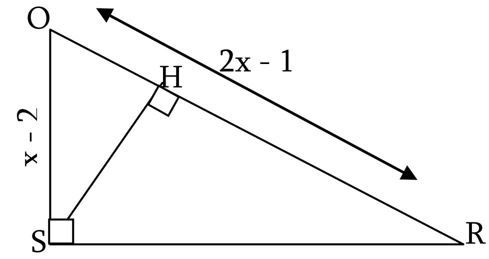
Consider the two polynomials:

$P(x) = 4(x-3)^2 - (x+1)^2$ and $Q(x) = (x-7)^2 - (x+1)(7-x) - 49 + x^2$

1) a) Show that P(x) can be written in the form of the remarkable identity $a^2 - b^2$ then factorise it.(1pt)
   b) Show that Q(x) = (x – 7) (3x + 1). (1pt)
   c) Verify that $Q\left(\frac{-1}{3}\right) = 0$ and justify the inequality $Q\left(-\frac{1}{3}\right) + 110 > P\left(-\sqrt{2}\right)$ (don't use a calculator). (1 ¼ pts)
2) a) Write P(x) in the form $ax^2 + bx + c$ where a, b, and c are integers to be determined. (1pt)
   b) Determine the values of the real numbers m and n so that the polynomial $A(x) = (2m-1)x^2 + (3n-2)x + 35$ is identical to P(x). (1pt)
3) Consider the fractional expression $F(x) = \frac{P(x)}{Q(x)}$.
   a) Precise the conditions for the existence of F(x), and say why it is not defined for any integer. (1pt)
   b) Simplify F(x), then solve the equation F(x) = -1. (1 ¼ pts)

<u>4th exercise: (4 ½ pts)</u>

Consider a right triangle SOR at S such that OS = x – 2 and RO = 2x – 1 as indicated in the figure to the right:



1) For what values of x, does OS exist? Justify. (½ pt)
2) a) Show that $SR^2 = 3(x^2 - 1)$. (¾ pt)
   b) Does there exist a value for x such that SR = 3? Justify (1pt)
   c) Show that the area of the triangle OSR is given by: $A(x) = \frac{(x-2)\sqrt{3(x^2-1)}}{2}$. (¾ pt)
   d) Write the area of triangle OSR in another way then deduce the length SH in case x = 3. (1pt)
   e) Write SH in scientific notation. (½ pt)

<u>5th exercise: (9pts)</u>

Consider a circle (C) of center O and radius R = 5cm. Designate by [AB] and [DE] two perpendicular diameters of this circle (C). C being the point of the arc AD such that AC = 5cm, and [Ax) is the semi-straight-line holding [AC].

1) Draw a figure. (½ pt)
2) Show that triangle ACO is equilateral and justify that triangle ACB is semi-equilateral. (1pt)
3) Show that $ACE = 45°$ then deduce that [CE) is the interior bisector of ACB and that [CD) is the exterior bisector of ACB . (1 ½ pts)
4) Let F be the orthogonal projection of B on (CD).
   a) Show that (CE) and (BF) are parallel. Deduce the nature of the triangle CBF.(1 ½ pts)
   b) Show that (OF) is the perpendicular bisector of [BC]. (1pt)
5) (FB) cuts the circle (C) at N.
   a) Knowing that the quadrilateral ACNB is cyclic, calculate CNB then deduce that $BCN = 15°$.(1pt)
   b) Calculate CD then show that $DNC = 15°$. Deduce that (DN) is parallel to (CB). (1 ½ pts)
6) (CE) cuts [AB] at M, Calculate AMC . (1pt)