ليسه دي زار

الشهادة المتوسطة　　　　　التجربة الأولى لعام 2011 – 2012

مسابقة في الرياضيات الانكليزي	المدّة : ساعتين	الإسم :	الرقم :

إرشادات عامة:

- يسمح بإستعمال ألة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على ألمسائل بالترتيب الذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 30
- عدد المسائل: 5

1st exercise: (6pts)

In the table below, only one among the proposed answers to each question is correct. Write down the number of each question and give, **with justification**, its correct answer.

No.	Questions	Answers		
		a	b	c
1.	The rational number 3.777… is inserted between	$\frac{11}{3}$ and $\frac{13}{3}$	$\frac{38}{9}$ and $\frac{43}{9}$	$\frac{31}{9}$ and $\frac{35}{9}$
2.	P(x) and Q(x) are two polynomials of same degree 3, then their product has a degree:	$3 \times 3 = 9$	$3^3 = 27$	$3 + 3 = 6$
3.	$A(x) = \sqrt{\frac{9}{x^2} + \frac{6}{x} + 1}$ where x < - 3 then A(x)=	$-\frac{x+3}{x}$	$\frac{x+3}{x}$	$\frac{3}{x} + 1 + \sqrt{\frac{6}{x}}$
4.	Given the two real numbers $x = \frac{1 + \frac{1}{2} \times \frac{4}{5}}{2 + \frac{4}{5}}$ and $y = \frac{\sqrt{18} + \sqrt{8}}{\sqrt{200}}$ then,	x = y	x < y	x > y
5.	A, B, and C are three points such that: $AB = \frac{6 + 4\sqrt{3}}{3 + \sqrt{3}}$, $AC = 2 + 2\sqrt{3}$, and $BC = 1 + \sqrt{3}$ then,	ABC is an isosceles triangle at B	ABC is a semi-equilateral triangle	B is the midpoint of [AC]

2nd exercise: (6pts)

Consider a triangle MON such that:

$NO = \sqrt{15} - \sqrt{3}$, $MN = \frac{2\sqrt{5} + 2}{3 + \sqrt{5}}$, and $MO = 3\sqrt{45} - 4\sqrt{20} - \sqrt{80} + \sqrt{125} - \sqrt{4}$ (unit of length is cm).

1) a) Write MO in the form $a\sqrt{5} + b$ where a and b are integers to be determined. (¾ pt)
   b) Rationalize the denominator of MN, then deduce that MO = 2MN. (1pt)
2) a) Develop and reduce MO^2, MN^2 and NO^2. (1 ½ pts)
   b) Deduce that triangle MON is semi equilateral. (1pt)
3) a) Show that the area of triangle $MON = \left(3\sqrt{3} - \sqrt{15}\right)cm^2$. (¾ pt)
   b) Compare the area of triangle MON to that of the rectangle ABCD of dimensions $\left(-2 + 2\sqrt{5}\right)$ and $\left(\sqrt{15} + \sqrt{3}\right)$. (1pt)

3rd exercise: (6 pts)

Consider the two polynomials:

$P(x) = (3x-a)^2 + (3a+x)x - (a-b)(a+b) - b(2x+b-1)$;

$Q(x) = 10x^2 - 7x + 1$

$E(x) = 8x^2 - 8x + 2 + (3-6x)(3x+2) - 5 + 20x^2$

1) a) Show that $P(x) = 10x^2 - (3a+2b)x + b$. (1pt)
   b) Calculate a and b so that P(x) and Q(x) are identical polynomials. (1pt)
2) Show that $Q(x) = (5x-1)(2x-1)$ (½ pt)
3) Write E(x) in the form of a product of two factors of the first degree. (1pt)
4) **Suppose in this part that $E(x) = (2x-1)(5x-3)$** . Consider the fractional expression $M(x) = \dfrac{Q(x)}{E(x)}$
   a) Determine the values of x for which M(x) is **not defined**. (¾ pt)
   b) Hussein says that M(x) is defined for any natural number. Is he right? Justify. (¾ pt)
   c) Simplify M(x), then solve the equation $M(x) = \dfrac{x-1}{x+2}$.(1pt)

4th exercise: (3 pts)

Given the following numbers:

$A = \frac{2}{3} - \frac{2}{3} \times \left(\frac{4}{5}\right)^{-1} \div \left(\frac{3}{2} - 1\right)$; $B = (2^{15} + 2^{16}) \div (2^{14} + 2^{16})$ and $C = \dfrac{0.48 \times (10^3)^4 \times 0.001}{0.3 \times 10^{-4} \times 100}$

1) Show that A is an integer. (1pt)
2) Show that B = 0.012×10^2 then justify if B is a decimal number. (1pt)
3) Write C in scientific notation. (1pt)

5th exercise: (9pts)

Given a circle (C) of center O and radius 6cm. [AB] is a diameter of the circle, (Δ) and (Δ') are two tangents to (C) at A and B respectively.

M is a point of the circle (C) such that $\widehat{MB} = 60^o$.

1) Draw a figure and justify how you placed point M. (1pt)
2) a) Calculate $M\hat{A}B$ then deduce that the triangle AMB is semi-equilateral. (1pt)
   b) Deduce the lengths of the sides [BM] and [AM]. (1pt)
3) The tangent to (C) at M cuts the tangents (Δ) and (Δ') at E and F respectively.
   a) Show that EF = AE + BF. (1pt)
   b) Show that $E\hat{O}F = 90^o$ by two methods:
      i) Without any calculation. (1pt)
      ii) With calculation. (1pt)
4) The straight-line (OE) cuts [AM] in I.
   a) Show that (OI) is parallel to (BM). (1pt)
   b) Prove that the point I belongs to a circle (C') of center O and radius $\dfrac{BM}{2}$. (1pt)
   c) Deduce that (AM) is tangent to circle (C') at a point to be determined. (1pt)