ليسه دي زار

الشهادة المتوسطة   التجربة الأولى لعام 2012 – 2013

مسابقة في الرياضيات الانكليزي	المدّة : ساعتين	الإسم :	الرقم :

إرشادات عامة:

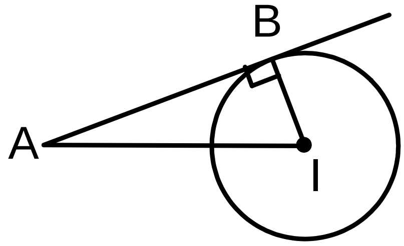
- يسمح بإستعمال ألة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على ألمسائل بالترتيب الذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 30
- عدد المسائل: 4

1st exercise: (6pts)

Decide if each statement is true or false and justify.

1) Let (C) be the circle of center W and radius R = 3 cm and E the point such that $WE = \frac{5\times 4^{13}\times 2^{11} - 16^{9}}{\left(3\times 2^{17}\right)^{2}}$, then E is interior to circle (C). **(1 ¼ pts)**
2) If $x=\sqrt{\left(2-\sqrt{2}\right)^{2}}$ and $y=\sqrt{\left(1-\sqrt{2}\right)^{2}}$ then $\frac{x}{y}=\sqrt{2}$. **(1 ¼ pts)**
3) Consider to the right the circle (δ) of center I and radius $IB=\frac{3}{2}$.

   (AB) is tangent to the circle at B such that $AB=\sqrt{4.69\overline{4}}$, then $IA=\frac{5\sqrt{10}}{6}$ cm. **(1 ½ pts)**



4) Given $a=-\frac{\sqrt{2+\sqrt{3}}}{2}$ and $b=\frac{\sqrt{2}+\sqrt{6}}{4}$ then a = - b. **(1pt)**
5) The polynomial $P(x)=(m^{2}-3)x^{3}+2x-1$ is for 3rd degree for all real values of m. **(1pt)**

2nd exercise: (5pts)

Consider the following numbers:

$$a=\left[\frac{7}{18}\times\frac{2}{7}-\left(1-\frac{5}{2}\right)^{-2}\right]^{2} \qquad , \qquad b=\frac{2-\sqrt{3}}{\left(5-2\sqrt{6}\right)\left(5+2\sqrt{6}\right)}$$

1) Show that $a=\frac{1}{9}$ and state with justification the nature of **a** , then write **a** in scientific notation. **(1¼pts)**
2) Rationalize the **numerator** of **b** and show that $b=\frac{1}{2+\sqrt{3}}$. **(1pt)**
3) Let $E(x)=x^{2}-4x+c$.
   a) Verify y that $b=2-\sqrt{3}$. **(½ pt)**
   b) Calculate **c** knowing that $2-\sqrt{3}$ is a root of E(x). **(1pt)**
   c) **Suppose in this part that c = 1** and $x=2+\sqrt{3}$. Compare x^{2} and 4x – 1 then deduce the roots of the equation $x^{2}-4x+1=0$. **(1 ¼ pts)**

3rd exercise: (9 ½ pts)

Consider the polynomial $f(x)=x^{3}+3-3x^{2}-x$

1) Determine with **justification** the domain of definition of f. **(¾ pt)**
2) a) Write f(x) in the form of a product of 3 binomials of 1st degree. **(1 pt)**
   b) Show that $f(x+1) = x(x^2-4)$, then factorise $f(x+1) - f(x-1)$. **(1 ½ pts)**
   c) Solve in the **set of natural numbers** the equation $f(x+1) = f(x-1)$. **(¾ pt)**
3) Determine the values of a, b, c, and d so that f(x) is identical to G(x) where $G(x) = (a-b)x^3 + 3(a+b)x^2 - 5d\,x + c^2 - 1$. **(1 ½ pts)**
4) Let $K(x)=\dfrac{f(x)}{(x+1)^{2}-(2x-4)^{2}}$.
   a) What do we call K(x)? **Justify**. **(¾ pt)**
   b) Calculate K(5) if possible. What can you say about x = 5? **(1pt)**
   c) Determine the values of x for which K(x) is not defined. **(1pt)**
   d) Simplify K(x) and solve the equation K(x) = (x +1). **(1 ¼ pt)**

4th exercise: (9 ½ pts)

(C) is a semi-circle of center O, radius **5<R<6 cm** and diameter [AB]. M is the point of (C) such that $M\hat{A}B=30^{o}$.

1) Draw a figure. **(½ pt)**
2) Calculate $M\hat{O}B$, and then deduce the nature of triangle MOB. **(1pt)**
3) The perpendicular drawn from O to (AB) cuts [AM] at K, and let E be the symmetric of B with respect to K.
   a) What does (OK) represent for [AB]? Deduce the nature of triangle KAB. **(1pt)**
   b) Show that (OK) and (AE) are parallel. Deduce the relative position of (AE) with respect to (C). **Justify**. **(1 pt)**
4) (BE) cuts (C) at F. Calculate AF in terms of R. **(1pt)**
5) [AF) and [BM) intersect at I. what does K represent for triangle IAB? Deduce that I, K, and O are collinear. **(1½ pts)**
6) The other tangent to the semi-circle (C) through E cuts:
   - (C) at G .
   - The tangent to (C) at B in J.

   Show that the triangle EOJ is right at O and that EJ = EA + JB. **(2 pts)**
7) a) Calculate the measure of arc $\overset{\frown}{AF}$ then deduce that (FM) and (AB) are parallel. **(¾pt)**
   b) What is the nature of the quadrilateral AFMB? **(¾pt)**