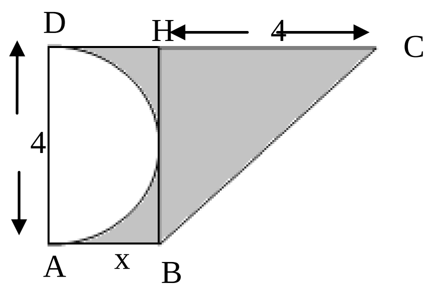
مسابقة في الرياضيات الانكليزي	المدّة : ساعتين	الإسم :	الرقم :

إرشادات عامة:

- يسمح بإستعمال آلة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على المسائل بالترتيب الذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 30
- عدد المسائل: 5

1st exercise: (6 ½ pts)

In the table below, only one of the proposed answers to each question is correct. Write down the number of each question and give, **with justification**, the answer corresponding to it.

№	Questions	Answers		
		A	B	C
1.	Let $A=\dfrac{2^n\left(3^{n+1}-3^n\right)}{6^{n+1}-6^n}$, then A =…	1	$\frac{5}{2}$	$\frac{2}{5}$
2.	Given the numbers : $X=\dfrac{9^{11}+9^{10}+9^9}{3^{20}+3^{19}+3^{18}}$. $Y=4\left(2+\sqrt{8}\right)^2\left(3-2\sqrt{2}\right)-2$. then	Y = 2X	X = 2Y	X = Y
3.	Consider the equation : $3x\sqrt{2}=\sqrt{2}-3x$. The solution of this equation is x=……..	$\frac{2-\sqrt{2}}{3}$	$\frac{1}{6}$	$\frac{2+\sqrt{2}}{3}$
4.	The number $H=\sqrt{\left(2.\overline{1}\right)^2-\frac{37}{81}}$ =....	$\frac{1}{9}$	9	2
5.	Let ABCD be a right trapezoid. (C) is a semi-circle of diameter [AD] and H is the orthogonal projection of Bon [DC] such that AB = x, AD = HC = 4cm.  If the area of the shaded region is equal to $20-2\pi$ cm^2 then x =…. …	$x=4\sqrt{2}$	$x=3\pi+3$	$x=3$

2nd exercise: (3pts)

Consider the two expressions: $X = \frac{1}{a+1} + \frac{1}{b+1}$ and $Y = \frac{1}{a} + \frac{1}{b}$ where a and b are real numbers.

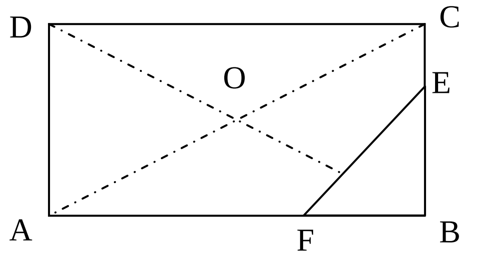
1) For what values of a and b is the expression X defined? **(¾ pt)**
2) a) In case X = 0 ,deduce that a + b = -2. **(¾ pt)**

   b) If $Y = \frac{1}{3}$ then show that ab = - 6. **(¾ pt)**

   c) Calculate $a^2 + b^2$. **(¾ pt)**

3rd exercise: (6 ½ pts)

Consider a rectangle ABCD of center O such that $AC = 4\sqrt{3}$ cm and $B\hat{A}C = 30^0$.



1) Show that the dimensions AB= 6 and AD= $2\sqrt{3}$. **(1½ pts)**
2) Let EFB be a triangle drawn inside the rectangle ABCD such that $EB = \frac{\sqrt{6}+\sqrt{2}}{2}$ and $FB = \sqrt{2+\sqrt{3}}$ as indicated to the right.

   a) Show that $\left(\sqrt{6}+\sqrt{2}\right)^2 = 4\left(2+\sqrt{3}\right)$, then deduce the simplest form of $\sqrt{\left(2+\sqrt{3}\right)}$. **(1½ pts)**

   b) Show that the triangle EFB is right isosceles. **(1½ pts)**

   c) Calculate $\boldsymbol{A_1}$, the area of triangle EFB, then deduce $\boldsymbol{A_d}$, the area of the domain included between rectangle ABCD and the triangle EFB ($\boldsymbol{A_d}$= area of AFECD) . **(1½ pts)**

   d) Calculate the approximate value of $\boldsymbol{A_d}$ to the nearest 0.001 by excess. **(½ pt)**

4th exercise: (5½ pts)

Consider the following polynomial: $P(x) = 3x^2 - 2x - 40$.

1) Show that P(x) gets the same value in the two following cases x = -3 and $x = \frac{11}{3}$.**(1½ pts)**
2) a) Develop and reduce (x + 3) (3x - 11). **(½ pt)**

   b) Deduce the solutions of the equation P*(x)* = - 7. **(¾ pt)**
3) Let *Q(x) = (m + n x)(x -3) + p be a polynomial* where m, n and p are real numbers.

   a) Show that Q*(x)* = $n\,x^2 + (m - 3n)x - 3m + p$. **(1pt)**

   b) Calculate *m, n and p* so that *P(x)* and *Q(x)* are identical. **(1pt)**
4) Solve the equation *Q(x)* = -40. **(¾ pt)**

5th exercise: (8 ½ pts)

Consider the angle $x\hat{I}y = 60^0$ and let [It) be the bisector of $x\hat{I}y$. E is the point on the bisector [It) such that IE = 4 cm. M and N are the orthogonal projections of E on [Ix) and [Iy) respectively.

1) Draw a figure. **(½ pt)**
2) Show that without any calculation IM = IN. **(¾ pt)**
3) Show that the points I, M, E and N belong to the same circle whose center and radius are to be determined. **(Don't draw this circle) (1 pt)**
4) (C) is the circle of center E and radius EM.

   a) Show that (IN) and (IM) are tangent to the circle (C) at N and M respectively. **(1½ pts)**

   b) Determine the nature of triangle IMN. **(1 pt)**

   c) (IE) cuts the circle (C) at P. **(P is between I and E)**. Show that P is the midpoint of the arc $\overset{\frown}{MN}$.**(¾ pt)**
5) J is the point diametrically opposite to P.

   a) Show that $M\hat{J}N = 60^0$ then deduce that triangle MNJ is equilateral. **(1½ pts)**

   b) Deduce that (MJ) and (IN) are parallel. **(¾pt)**

   c) Show that IMJN is a rhombus. **(¾ pt)**