مسابقة في الرياضيات الإنكليزي	المدّة : ساعتان	الإسم :	الرقم :

إرشادات عامة:

- يسمح بإستعمال ألة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على ألمسائل بالترتيب الذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 30

1st exercise: (7¼ pts)

State **with justification** whether each of the following statements is ***true*** or ***false***:

1) Given the real number $A=\sqrt{\frac{9^8-3^6}{9^7-3^4}}$, then $A=3$. **(1 pt)**

2) If (C) is a circle of center O and radius $r=\frac{2\sqrt{3}+3\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ cm and (d) is a straight -line at a distance $\sqrt{6}$ cm from the center O, then (d) is exterior to the circle (C). **(1½ pts)**

3) The solution of the following equation $x\sqrt{2}=\sqrt{2}+2$ is $x=\sqrt{2}$. **(1 pt)**

4) The third of 3^{90} is 3^{30}. **(¾ pt)**

5) $P(x)=(2mx-3)(x+1)-(x-3)(x+m)$ is a second degree polynomial in x for all values of $m\neq\frac{1}{2}$. **(1pt)**

6) If $E=(5-\sqrt{2})^2+2(1+5\sqrt{2})$, then E is an irrational number. **(1 pt)**

7) If $C(O,r)$ is a fixed circle of diameter $[AB]$, M is any variable point on (C), and I is the midpoint of $[AM]$, **then the set of points I is the perpendicular bisector of $[AM]$. (1 pt)**

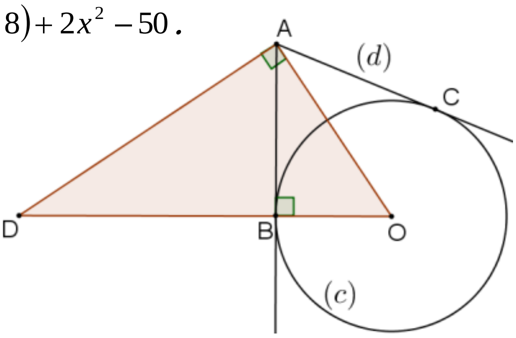
2nd exercise: (10 pts)

Consider the following polynomials:

$E(x)=(2x-1)^2-(x+4)^2$ and $F(x)=2x^2-20x+50-(10-2x)(x+8)+2x^2-50$.

Part A :

In the adjacent figure $[AB]$ is the height issued from the vertex of the right triangle ADO of hypotenuse $[OD]$, and (C) is a circle of center O and radius OB = 6 cm. From A we draw a straight line (d) that cuts (C) at point C.



Given: $AD=2x-1$ and $DB=x+4$. $(x>2)$

1) Use triangle ABD to prove that: $AB^2=E(x)$. **(1 pt)**
2) Write $E(x)$ in the form ax^2+bx+c ***where a, b & c are integers to be determined.*** **(1¼ pts)**
3) a) Develop and reduce the expression: $(x+4)\ (3x-24)$. **(¾ pt)**

   b) Use **part 1** to calculate the value of x if $AB=9cm$. **(1 pt)**
4) Suppose that $B\hat{A}O=30^\circ$, calculate the lengths of **the segments $[AO]$ and $[BO]$. (1½ pts)**

Part -B :

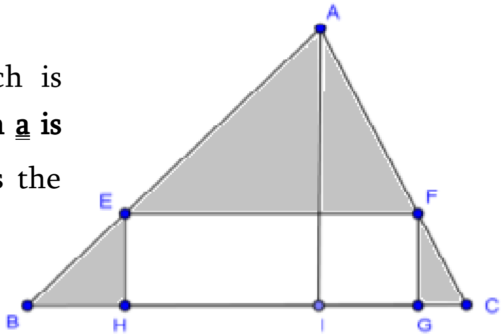
In this part suppose that : $AC^2=F(x)$

1) Factorize: ***E (x).*** **(1 pt)**
2) Prove that : $F(x)=2(x-5)(3x+8)$. **(1½ pts)**
3) Solve the equation : $E(x)=F(x)$. **(1 pt)**
4) *We admit that (d) is the second tangent issued from from A to (C)* , then calculate the value of x. **(1pt)**

3rd exercise: (5½ pts)

(The unit of length in this exercise is *the cm*)

In the adjacent figure the area of the rectangle $EFGH$ which is inscribed in the triangle ABC is: $A = 10 + 4\sqrt{6}$ Where, its **length $\underline{\underline{a}}$ is double its width $\underline{\underline{b}}$** and $BC = 4\sqrt{10}$, $AI = \sqrt{10} + 2$ where I is the orthogonal projection of A on [BC] .



1) a) Develop : $\left(\sqrt{2}+\sqrt{3}\right)^2$. **(½ pt)**

   b) Prove that: $b = \sqrt{2}+\sqrt{3}$.**(1 pt)**

2) If P is the perimeter of rectangle $EFGH$, then prove that $P = m\left(\sqrt{2}+\sqrt{3}\right)$ where ***m*** **is an integer** to be determined. **(1 pt)**

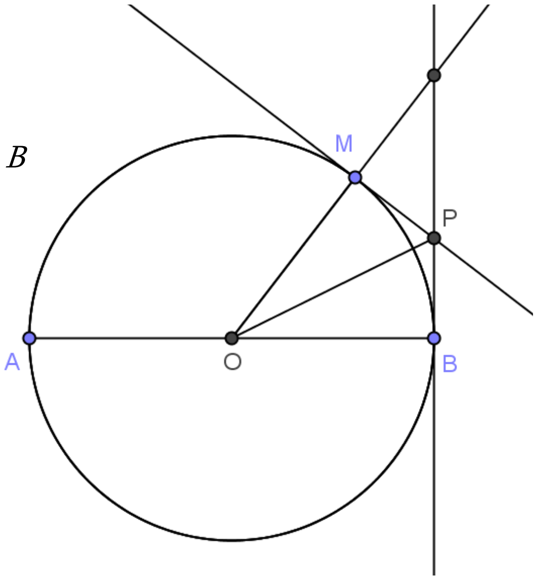
3) Calculate the area of triangle ABC, then deduce the area of the shaded part. **(1½ pts)**

4) If $p(x) = ax^3 - 2bx^2 - x^3 + x^2 + cx + d + 1$ and $q(x) = (3x+2)(x+1)$ are the dimensions of the rectangle $EFGH$ then for what values of a, b, c and d is $EFGH$ a square. **(1½ pts)**

4th exercise: (7¼ pts)

Given in the adjacent figure:

- (C) is a circle of center O and diameter $AB = 6\ cm$..
- M is a point on (C) distinct from A & B.
- The tangent drawn from M cuts the tangent issued from B at point P and the straight line (AB) in Q.
- The straight line (AM) cuts (BP) in R.
- The straight line (OM) cuts (BP) in S.



1) Reproduce and complete the figure. **(¾pt)**
2) Prove that (OP) is parallel to (AM). **(1 pt)**
3) Prove that P is the midpoint of $[BR]$. **(1 pt)**
4) Prove that:
   a) (OP) is perpendicular to (QS). **(1 pt)**
   b) (AS) is perpendicular to (QR). **(1 pt)**
5) Consider M to be a variable point on (C) and J be the symmetric of B with respect to M
   a) Prove that the triangle ABJ is isosceles. **(1 pt)**
   b) Determine on which line the point J varies as M describes the circle (C). **(1½ pts)**