مسابقة في الرياضيات الإنكليزي	المدّة : ساعتان	الإسم :	الرقم :

إرشادات عامة:

- يسمح بإستعمال ألة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على ألمسائل بالترتيب االذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 25

1st exercise : (4½ pts)

In the following table, **just one** of the proposed answers is correct. Indicate the number of the question and its corresponding answer **and justify.**

№	Questions	Answers a	b	c
1.	If a is a real number such that : $a=\sqrt{4.69\overline{4}}+\frac{1}{12}$, then $a^{-1}=\dots$	3^{-n}	$\frac{4}{9}$	$\frac{43}{9}$
2.	If $a=\frac{1}{2^{10}}$, $b=2^5$ and $c=\frac{1}{2^5}$ and if $x=\frac{1}{b(a+c)-1}$, then x = ...	2^5	2^{-5}	$2^{-5}+1$
3.	If $C=\frac{\sqrt{252}}{\sqrt{112}}-\frac{\sqrt{45}}{\sqrt{245}}+\frac{5}{4}$, then $C=\dots$	$\frac{65}{28}$	2	$\frac{13}{8}$
4.	Let ABC be a triangle such that: BC = $\sqrt{\frac{4(3+\sqrt{5})}{3}}$, AB = $\sqrt{\frac{5}{3}}+\frac{\sqrt{3}}{3}$ and AC = $\frac{\sqrt{15}+\sqrt{3}}{3}$, then the triangle ABC is …	Isosceles	Right isosceles	equilateral

2nd exercise: (4½ pts)

Let $ABCD$ be a quadrilateral such that: **(The unit of length is the cm).**

$$AB=\sqrt{2.25}+\sqrt{\frac{64}{9}}\ ;\ BC=\sqrt{12.25}\times\frac{10}{21}\times\sqrt{\frac{3}{2}}\ ;\ CD=\sqrt{7.\overline{3}-\frac{61}{12}\times\left(\frac{5}{3}\right)^2}\ \text{and}\ AD=\left(\frac{1}{2}-\frac{1}{3}\times\frac{1}{4}\right)\left(\sqrt{2}\times\sqrt{12}\right).$$

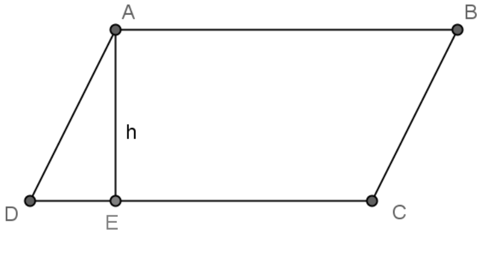
1) a) Reduce, **then say if each of the given numbers** is either **rational or irrational,** and **justify your answer. (2 pts)**

b) Deduce the nature of $ABCD$. **(¾ pt)**

2) Let E be the foot of the height issued from A to $[DC]$.

h is the length of $[AE]$ ($AE=h$), such that :

$$h=\sqrt{\left(2\sqrt{2}-4\right)^2}+\sqrt{\left(\sqrt{2}-1\right)^2}-\sqrt{\left(\sqrt{2}-2\right)^2}.$$



a) Show that h is an integer. **(1 pt)**

b) Calculate the area of the quadrilateral $ABCD$. **(¾ pt)**

3rd exercise : (6 pts)

Consider the following algebraic expressions:

$E(x)=(m+1)x^3-3x^2-(m+3)x+(2m+1)$,

$G(x)=(4x^2-4x+1)+(6x-3)(x+2)-5+20x^2$, and

$F(x)=(a+x)^2-(x^2+1)(b+2x)+a(x^3-a)+c(x-1)-1$.

(a, b, c and m are four real numbers and m is a parameter)

1) Calculate *m so that* 1 a root of $E(x)$. **(1 pt)**

For the rest of the exercise, consider: $E(x)=2x^3-3x^2-4x+5$.

2) Show that $E\left(\sqrt{2}\right)$ is an integer. **(1pt)**

3) a) Show that : $F(x)=(a-2)x^3+(1-b)x^2+(2a-2+c)x-b-c-1$. **(1½pts)**

b) Calculate a, b and c so $E(x)$ and $F(x)$ are identical. **(1pt)**

4) Factorize G(x) then deduce its roots. **(1½ pts)**

4th exercise: (10 pts)

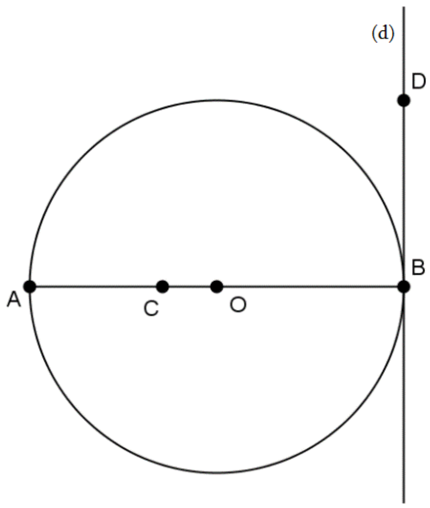
In the following figure we have:

* (C) is a circle of center O, of radius R and of diameter [AB].

* C is a point on [AO] such that: $AC=\frac{2}{3}R$.

* (d) is the perpendicular to (AB) at B.

* D is a point on (d) such that: $D\hat{O}B=60^o$.



1) Reproduce the figure. **(¾ pt)**

2) a) Show that : $BC=\frac{4}{3}R$. **(½ pt)**

b) Determine the nature of the triangle DOB, and then calculate DB in terms of R. **(¾ pt)**

c) Calculate, in terms of R, the area of the triangle COD. **(¾ pt)**

3) Let M be the orthogonal projection of B on [CD]. The perpendicular bisector of [AB] cuts [CM] at N. Show that M, N, O and B belong to the same circle of center I and of diameter to be precised. **(¾ pt)**

4) The tangent to (C) issued from D other than (d), touches the circle at E, (E is the point of tangency), and it cuts the line (AB) at P.

a) What does [DO) represent for the angle $E\hat{D}B$ **? Justify. (½ pt)**

b) **Using the triangle BDP**, show that: $DP = 2\times BD$. **(¾ pt)**

c) Deduce that E is the midpoint of [DP]. ***(¾ pt)***

5) **F is the symmetric of E with respect to (AB).**

a) Show that AEOF is a rhombus. **(¾ pt)**

b) Deduce that the points F, O and D are collinear. **(¾ pt)**

c) Show that (PF) is tangent to (C) at F. **(¾ pt)**

6°) ***In this part, we suppose that D varies on the straight line*** (d).

a) Indicate the fixed points among O, M, N, D and B. **(¾ pt)**

b) What is the locus of **I, the midpoint of [BN]? Justify. (¾ pt)**

c) Let K be the intersection point of [DO] and [EB].
On **which line** does K vary? **Justify. (¾ pt)**