مسابقة في الرياضيات الإنكليزي	المدّة : ساعتان	الإسم :	الرقم :

إرشادات عامة:

- يسمح بإستعمال ألة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على ألمسائل بالترتيب االذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 20

1st exercise: (4 pts)

In the following table, **just one** of the proposed answers is correct. Indicate the number of the question and its corresponding answer **and justify.** (1 pt – 1½ pts – ½ pt - 1pt)

No	Questions	Answers a	Answers b	Answers c
1.	If p and q are two real numbers such that: $p \times q = \dfrac{1+\frac{1}{2}}{3}$ and $p+q = \dfrac{\sqrt{48}}{\sqrt{27}} + \dfrac{2}{3}$, then: $p^2+q^2 = \ldots$	- 3	3	5
2.	If $E = \dfrac{1}{5} - \left(\dfrac{2}{5}\right)^2$, $F = \left(2-\sqrt{5}\right)^2 + 2\left(8+\sqrt{20}\right)$ And $G = \sqrt{108} - 3\sqrt{12} - 5\sqrt{25}$, then **the two reciprocal numbers** are …	E and F	E and G	F and G
3.	If « 3 » **is a root** of the polynomial $P(x) = (a+1)x^2 + (x-3)(x+5)$, then $a = \ldots$	0	1	- 1
4.	If C (O ; r) and C'(O' ; r') are two circles **externally tangent at A**, and (d) is a common tangent to (C) and (C') that cuts them, respectively, at M and N, and cuts the interior common tangent at T, then the triangle AMN is …………	Right	isosceles	equilateral

2nd exercise: (4 pts)

Given the following polynomials: $P(x) = (3x-1)(x-5) - x^2 + 25$ and $Q(x) = P(x) + 34 - x^2$

1) Expand and reduce $P(x)$, and show that: $P(x) = 2x^2 - 16x + 30$. (¾ pt)

2) Factorize $P(x)$, and then deduce its roots. (½ pt)

3) a) Show that: $Q(x) = (ax+b)^2$, where « a » and « b » are 2 integers are to be determined. (¾ pt)

b) Solve the equation: $Q(x) = 1$. (½ pt)

4) Consider the fractional expression: $F(x) = \dfrac{(x-8)^2}{(2x-16)(x+1)}$.

a) Determine the values of x for which $F(x)$ is defined, and then simplify it. (¾ pt)

b) Solve the equation: $F(x) = 2$. (¾ pt)

3rd exercise : (7pts)

The three parts of this exercise are independent:

Part A: Given the triangle ABC such that:

$$AB = \left(\sqrt{\sqrt{11}}\right)^2 \; ; \; BC = 3\sqrt{3}\left(\sqrt{5}-\sqrt{3}\right)+\sqrt{\left(\sqrt{15}-9\right)^2} \text{ and } AC = \sqrt{\sqrt{50}-1}\times\sqrt{\sqrt{50}+1}$$

1) Show that: $BC = 2\sqrt{15}$. (¾ pt)
2) Prove that ($AC \times \sqrt{49}$) is the **square of a prime number** to be determined. (¾ pt)
3) Deduce that ABC **is a right triangle** whose hypotenuse is to be determined. (¾ pt)

Part B: Given the three numbers:

$$X = 5+\sqrt{48}+\frac{3}{2}\sqrt{24}-3\sqrt{6} \; ; \; Y = \frac{3\sqrt{3}\times\sqrt{9}}{\left(\sqrt{\frac{1}{\sqrt{3}}}\right)^2} \text{ and } Z = \frac{22}{5-\sqrt{3}}$$

1) Reduce X, and show that Y **is the cube of 3**. (1¼ pts)
2) Rationalize **the denominator** of Z. (¾ pt)
3) Deduce that: $(Z - X)^2 = Y$. (½ pt)

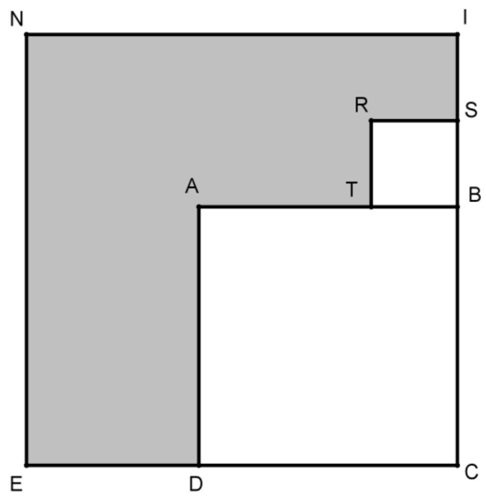
Part C: The adjacent figure shows:

NICE, ABCD and RSBT are **three squares** such that:

* $AB = \dfrac{3\sqrt{7}}{4-\sqrt{7}}+\dfrac{2-4\sqrt{7}}{3}$ cm.

* $NI = \sqrt{1,\bar{8}-0,\bar{1}}\times\left(\dfrac{4}{9}\right)^{-1}+2$ cm.

* RS = x cm (x is a real number such that $0 < x < 2$)

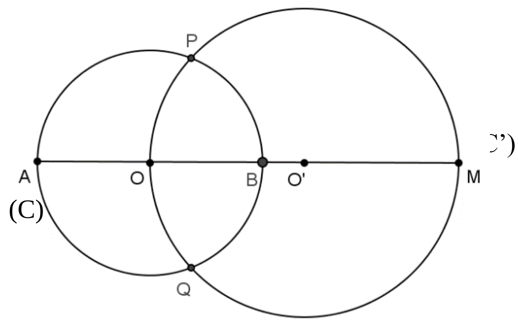


1) Show that: AB = 3 cm and NI= 5 cm. (1½ pts)
2) Express, in terms of x, the area of the **shaded domain.** (¾ pt)

4th exercise: (5 pts)

In the adjacent figure, we have:

* (C) is a circle of center O and of diameter [AB].
* (C') is a circle of center O' and of diameter [MO].
* M is a point on (AB) out of (C).



1) Reproduce the figure. (½ pt)
2) a) Determine the natures of the triangles OMP and OMQ. (¾pt)
   b) Deduce that the lines (MP) and (MQ) are tangent to (C) at P and Q, respectively. (1pt)
3) What does the line (OO') represent to [PQ]? **Justify.** (½ pt)
4) a) Show that the triangle OPQ is isosceles and deduce that: $\widehat{POQ} = 180° - 2\widehat{OPQ}$. (1 pt)
   b) Knowing that: $\widehat{OPM} = \widehat{OPQ} + \widehat{QPM}$, show that: $\widehat{POQ} = 2 \times \widehat{QPM}$. (¾ pt)
   c) Deduce that M is the midpoint of the long arc $\overset{\frown}{PQ}$. (½pt)