| مسابقة في الرياضيات الإنكليزي | المدّة : ساعتان | الإسم : | الرقم : |
|---|---|---|---|

**إرشادات عامة:**
- يسمح بإستعمال ألة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على ألمسائل بالترتيب االذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 30

## 1ˢᵗ exercise: (5.5pts)

In the following table, **just one** of the proposed answers is correct. Indicate the number of the question and its corresponding answer **and justify.** (1pt – 1pt– 1.5pts - 1pt- 1pt)

| No | Questions | Answers | | |
|---|---|---|---|---|
| | | **a** | **B** | **c** |
| 1. | Given $A = \frac{3-8a+9b}{a-4b}$ with $a \neq 4b$. If $a = \frac{1}{2}$ and $b = \frac{1}{3}$ then $A =$ | 2.4 | $\frac{-3}{5}$ | -2.4 |
| 2. | If the reciprocal of $\sqrt{2}$ is a root of $S(x) = (3x - a)(2x + a)$ then $a =$ | $\frac{3\sqrt{2}}{2}$ or $-\sqrt{2}$ | $\frac{-3\sqrt{2}}{2}$ or $\sqrt{2}$ | $\frac{3\sqrt{2}}{2}$ |
| 3. | If $M = \frac{4^{n+1} - 4^n}{2^{n+1} - 2^n}$ &P= $\left(\frac{14}{25} \times \frac{60}{21} + 5^{-1}\right) \times \left(\frac{3}{5}\right)^{-1}$ ,then | $M = P \times 2^n$ | $\frac{M}{P} = 2^{-n}$ | None |
| 4. | The number $H = \sqrt{(2.\bar{1})^2 - \frac{37}{81}}$, then $H =$ | $\frac{1}{9}$ | 9 | 2 |
| 5. | If $A(x) = \sqrt{\frac{16}{x^2} + \frac{8}{x} + 1}$ where x <- 4 then A(x)= | $-\frac{x+4}{x}$ | $\frac{x+4}{x}$ | $\frac{4}{x} + 2\sqrt{\frac{2}{x}} + 1$ |

## 2ⁿᵈ exercise: (14pts)
### The following parts are underlined independent
Part-A: Given the triangle **ABC** such that :

$$AB = \frac{4^8 - 8^6}{128^2 - 16^4} \; ; \quad BC = \left(\frac{1}{2}\right)^2 - \frac{3}{2^2} + \frac{3^2}{2} \quad \text{and} \quad AC = \left(\sqrt{7 - 2\sqrt{10}} \times \sqrt{7 + 2\sqrt{10}}\right)\left(\frac{\sqrt{8} \times 2\sqrt{27}}{3\sqrt{54}}\right)$$

Show all steps of calculations

1) a. Show that **AB=BC**. (1½pts)

   b. Show that AC is a square of 2. (1pt)

   c. Deduce the nature of the triangle ABC and draw it. (¾pt)

2) a. Let (AH) be the height issued from A and G be the centroid of triangle ABC. Calculate the lengths of AH and HG. (1½pt)

   b. Deduce the area of triangle HGC. (¾pt)

PartB: In the adjacent figure :

- ABCD is a square of side $AB = x - 6\ cm$.
- MBPN is a square of side $MB = 1\ cm$.

(x is a real number expressed in cm so that x > 6)

1. Express, in terms of **x**, the area of the **shaded domain.** (1pt)
2. Calculate **x** if the area of the shaded domain equals 3 cm². (1pt)
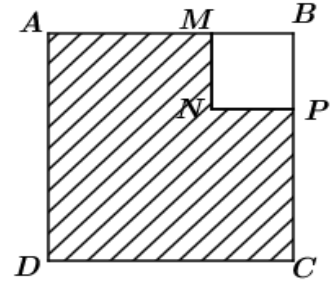
Part C:

In the orthonormal system of axes $(x'Ox\ \&\ y'Oy)$, consider the points $A(2;3), B(2;-3)\ \&\ C(n;3)$

1) Prove that the point $C$ is on the $y-axis$ if $n = \sqrt{(s+w)^2} + s + w$, where $(s+w) < 0$ (¾pt)

2) a. Place the points A,B & C in the given system. (1pt)
   b. Determine the nature of the triangle $ABC$ .(1pt)
   c. Let $(e)$ be the circle circumscribed about the triangle $ABC$ .

   Determine the coordinates of $I$ the center of $(e)$, and prove that the radius of $(e)$ $is\ \sqrt{10}$ . (1.5pts)

3) Let $F$ be any point in the plane so that, $IF = \sqrt{19 - 6\sqrt{10}}$

   a. Expand $(3 - \sqrt{10})^2$ then write $IF$ in form of one radical only. (1.5pts)
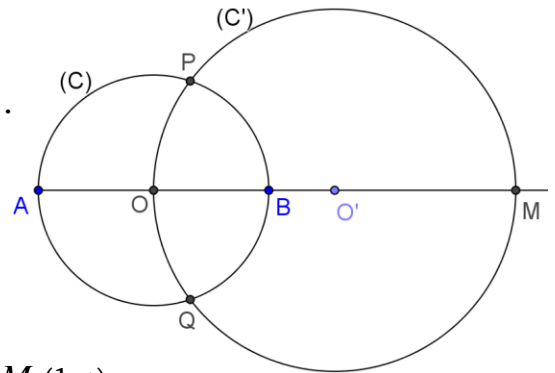   b. Deduce the relative position of $F$ with respect to $(e)$ .(¾pt)

3rd exercise: (4pts)

Given the following polynomials: $P(x) = (x+1)(x+2)(x-3) + 3(x+2)$ and $E(x) = (x-2)(2x+4)$

1. Show that $P(x) = x(x-2)(x+2)$ (1pt)
2. Let $A(x) = P(x) - E(x)$.
   a. Factorize $A(x)$ .(1pt)
   b. Solve the equation $A(x) = 0$ .(¾pt)

3. Suppose in this part that $A(x) = (x-2)(3x-1)$, and let $M(x) = \dfrac{P(x)}{A(x)}$

   a. What does $M(x)$ represent? (0.5pts)
   b. Determine the values of x for which $M(x)$ is not defined. (¾pt)

<u>4<sup>th</sup>exercise: (6.5pts)</u>

- $(C)$ is a circle of center $O$ and diameter $[AB]$.
- $M$ is a point of $(AB)$ and outside the circle $(C)$.
- $(C')$ is another circle of center $O'$ and diameter $[OM]$.
- $(C)$ and $(C')$ intersect at $P \& Q$.



1) Justify that $O\hat{P}M = 90°$ then deduce that $(MP)$ is tangent to $(C)$ at $P$.(1pt)

2) a. What is the nature of triangle $AOP$ ? (0.25pts)

   b. Use the rules of arcs and angles to show that $O\hat{P}A = B\hat{P}M$ (1pt)

3) a. What is the relative position of $(OO')$ with respect to $[PQ]$. (0.75pt)

   b. Use the triangle $MPQ$ to prove that $M$ is the midpoint of the major arc $PQ$ of $(C)$ (1pt)

4) Suppose in this part that $O\hat{M}Q = 30°$ and $O'M = 6cm$

    a) Calculate $MQ$ (1pt)

    b) Deduce the perimeter of the triangle $MPQ$ (1.5pts)

*Good Work*