ليسه دي زار | الشهادة المتوسطة | التجربة الأولى لعام 2018 - 2019

مسابقة في الرياضيات الإنكليزي	المدّة : ساعتان	الإسم :	الرقم :

إرشادات عامة:

- يسمح بإستعمال ألة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على ألمسائل بالترتيب االذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 20

1st exercise: (5¾pts)

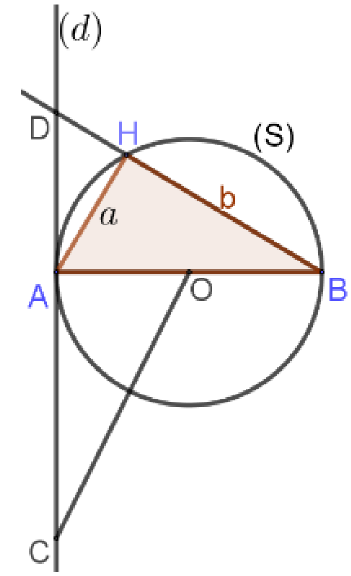
Part A: (2¾pts)

Consider in the adjacent figure:

- (S) is a circle of center O and diameter $[AB]$
- (d) is the tangent to (S) at point A and C is a point on (d) such that:

$$OC = \sqrt{\frac{81^2 \times 3^3}{9^4}} - \frac{\sqrt{8^2+6^2+8}}{2} + \sqrt{(\sqrt{37}-1)(\sqrt{37}+1)}\ cm \quad \text{and}$$

$$AC = \left(\sqrt{\frac{3}{2}} + \frac{1}{\sqrt{2}}\right)^2 - 2(1-\sqrt{3})cm$$



1) Simplify OC and show that $OC = 6cm$. **(1pt)**
2) Verify that $AC = 3\sqrt{3}$cm. **(1pt)**
3) Let R be the radius of the circle (S), where $2 < R < 6$
   Calculate the numerical value of R. **(¾ pt)**

Part B: (3pts)

In this part, let H be a point on the circle (S) of diameter $[AB]$ such that $AH = a$ & $BH = b$, where a & b are two strictly positive numbers. $[BH)$ cuts the line (d) at point D. **Take** $AB = 6cm$

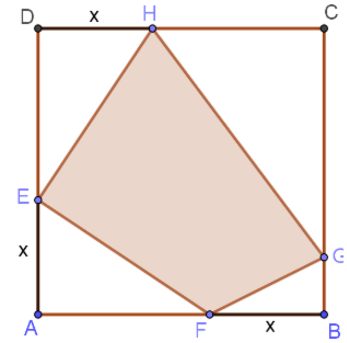
1) If $a = \frac{49^4 + 5\times 7^9}{7^8 \times 9} - 1$ then show that a is a **multiple of 3**. **(¾pt)**
2) a) Determine the exact nature of triangle ABH then calculate the measure of the arc $\widehat{BH}$. **(1¼pts)**
   b) **Show that b =** $3\sqrt{3}$ **then frame b** between two consecutive **integers**, and find its approximate value to the nearest 10^{-2} **by excess. (1pt)**

2nd exercise: (7¾pts)

Part A: (4¾pts)

Given the two polynomials: $A(x) = (2x-1)(x+1) - x^2 + x - 2 \quad and \quad B(x) = (x+3)(x-2) + 2x(x+3)$

1) Develop and reduce $A(x)$. **(¾pt)**
2) Prove that $A(x)+4$ is the square of a binomial. **(½pt)**
3) Show that $A(x) = (x-1)(x+3)$. **(¾pt)**
4) Factorize $B(x)$. **(½pt)**
5) Consider the rational expression $F(x) = \frac{A(x)}{B(x)}$
   a) For what values of x is $F(x)$ defined? Justify. **(½pt)**
   b) Simplify $F(x)$ then calculate $F(\sqrt{2})$ **(1pt)**
   c) The equation $F(x) = \frac{4}{11}$ admits **no solution** in the domain of $F(x)$. Justify. **(¾pt)**

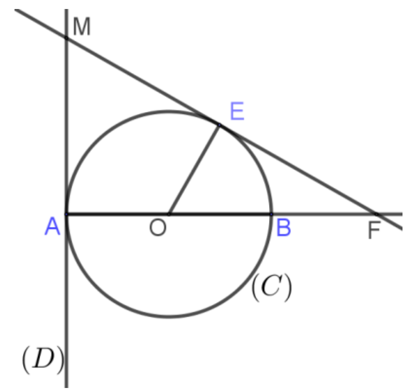


Part B: (3pts)

In the adjacent figure, $ABCD$ is a square of side $6cm$, where $AE = DH = BF = x$ and $BG = 1cm$. ($0 < x < 6$)

1) Show that the area of the shaded domain $FGHE$ is expressed by: $N(x) = x^2 - 4x + 21$. **(1 ¼pts)**
2) **Use part A (3)**, to find the values of x, so that $N(x) = -6x + 24$. **(¾pt)**
3) Consider in an orthonormal system x'Ox and y'Oy the point $A(N(x);2)$. Calculate x so that A is the symmetric of O with respect to the point $K\left(\frac{21}{2};1\right)$ **(1pt)**

3rdexercise: (6½pts)



In the adjacent figure:

- ✓ E is any point on circle (C) of center O, radius R and diameter $[AB]$.
- ✓ (D) is tangent to (C) at A.
- ✓ The tangent to (C) at E cuts (D) at M and $[AB)$ at F.
- ✓ $[EO)$ cuts (D) at G

1) Reproduce the figure, and place the point S, **the orthogonal projection** of O on $[FG]$. **(1pt)**
2) a) What does $[MO)$ represent for $A\widehat{M}E$? Justify. **(½pt)**

   b) **[OM) cuts the circle (C) at I.**

   What is the relative position of I with respect to arc $\widehat{AE}$? **(½pt)**
3) Prove that O is the **orthocenter** of triangle MGF, then deduce that the points M, O & S are collinear. **(1¼pts)**
4) Show that the points O, E, F & S belong to the same circle, whose center and diameter are to be precised. **(1pt)**
5) Prove that the triangle MGF is isosceles. **(¾pt)**
6) **Suppose that** $A\widehat{M}E = 60^o$.

   a) **We admit** that AM = $R\sqrt{3}$. Prove that the area of triangle AME is $\frac{3R^2\sqrt{3}}{4}$. **(¾pt)**

   b) Determine the nature of the quadrilateral AOEI. **(¾pt)**

Good Work