الثهادة المتوسطة

| الرقم : | الإسم : | اللدّة : ساعتين | مسابقة في الرياضيات الإنكليزي |
| :---: | :---: | :---: | :---: |

$1^{\text {rst }}$ exercise: (2pts)

## Solve then choose the correct answer

| No. | Questions | Answers |  |  |
| :--- | :--- | :---: | :---: | :---: |
|  |  | A | B | C |
| a | $(5+4)^{2}=$ | $5^{2}+4^{2}$ | $3^{4}$ | $-9^{2}$ |
| b | The half of $2^{2002}$ is | $2^{1001}$ | $2^{2001}$ | $1^{2002}$ |
| c | $5-2 \sqrt{6=}$ |  |  |  |
| d | $\sqrt{(2-\sqrt{5})^{2}}$ | $3 \sqrt{6}$ | $(\sqrt{6}-3)^{2}$ | $(\sqrt{2}-\sqrt{3})^{2}$ |

## $2^{\text {nd }}$ exercise: ( $2^{1 / 2} \mathrm{pts}$ )

A. Give the numbers

$$
B=\frac{2-\frac{1}{3}}{\left(\frac{1}{2}\right)^{2}} \quad C=\frac{4 \times 10^{-2} \times(+5) \times 10^{7}}{3 \times 10^{5}} \quad D=\frac{(3+\sqrt{11})^{2}-6 \sqrt{11}}{3}
$$

by showing all steps of calculation show that $\mathrm{B}=\mathrm{C}=\mathrm{D}$
B. 1) Let n be a non-zero natural integer. Prove the equality: $\frac{1}{n}-\frac{1}{n+1}=\frac{1}{n(n+1)}$
2) Use the preceding equality to calculate the expression:

$$
E=\frac{1}{1 \times 2}+\frac{1}{2 \times 3}+\frac{1}{3 \times 4}+\ldots \ldots+\frac{1}{7 \times 8}+\frac{1}{8 \times 9}+\frac{1}{9 \times 10}
$$

## $3^{\text {rd }}$ exercise: (4pts)

A. Given the following expressions.

$$
A(x)=x^{2}(4 x-4)-(3 x-4)(1-x)-3(x-1)^{2}
$$

and $B(x)=2 x^{2}-10 x+4$

1. Factorize $A(x)$ and prove that $A(x)=(x-1)(2 x+1)(2 x-1)$
2. Let $\mathrm{D}(\mathrm{x})=\mathrm{B}(\mathrm{x})-\mathrm{x}+1$

$$
\text { prove that } \quad D(x)=(x-5)(2 x-1)
$$

B. Given the polynomial

$$
P(x)=3 x^{2}-2 x-33
$$



1. Calculate the value of $P(x)$ for $x=-3$
2. Prove that $\frac{11}{3}$ is a root of $P(x)$
3. Expand the expression $(x+3)(3 x-11)$, then solve the equation $P(x)=0$
4. Find the area of the shaded part interns of $x$.
5. Find x such that this area equal $18 \mathrm{~cm}^{2}$.

## $4^{\text {th }}$ exercise: ( 2 pts )

The unit of length is the centimeter. Given a rectangle of dimension:
$x=2 \sqrt{108}-5 \sqrt{12}+2 \sqrt{32}-8 \sqrt{2}+\sqrt{16}$
$y=2 \sqrt{64}-8+4 \sqrt{75}-6 \sqrt{27}$

1. Simplify x and y .
2. State the length and the width of this rectangle. Justify.
3. Express the area of this rectangle in the form $a+b \sqrt{3}$
4. Find z such that $z=(x+4-4 \sqrt{3}) y$

## $5^{\text {th }}$ exercise: ( $4^{1 / 2} \mathrm{pts}$ )

In an orthonormal system of axis x'ox, y'oy. Given the straight line (D) $y=x+2$ and the points $E(0 ;-4)$ and H(-1, 1)

1. Plot (D) and prove that H belong (D).
2. (D) Cuts x'ox at F and y'oy at G. Prove by calculation that H is the midpoint of [FG].
3. Find the equation of $\left(D_{1}\right)$ parallel to $(D)$ and passes through $E$.
4. Let $B$ be the point of intersection between $\left(D_{1}\right)$ and $x$ 'ox and $J$ is the midpoint to [EB]. Find the equation of $(\mathrm{OH})$, and deduce that $\mathrm{H}, \mathrm{O}$ and J are collinear.
5. Find the equation of $\left(D_{2}\right)$ which is perpendicular to $(D)$ and passes through $A(2,7)$.

## $6^{\text {th }}$ exercise: (5pts)

In the adjacent figure given the circle $C(0 ; 2 \sqrt{3} \mathrm{~cm})$ of diameter [AB]; Let $E$ be a point on (C) such that $A B E=30^{\circ}$, the tangent at $A$ to $(C)$ cut $(B E)$ at $F$, and the tangent drawn from $F$ cuts $(C)$ at $P$.

1. Reproduce the figure.
2. Calculate AE and BE .
3. Prove that $\mathrm{BF}=8 \mathrm{~cm}$, deduce the length of AF .
4. The perpendicular to ( AB ) drawn from O cuts $[\mathrm{BF}]$ at M .
a. Find the nature of OMFA.
b. Calculate the midline of OMFA.
5. In this part, suppose that F is a variable point, (OF) cuts (AP) at I , Find the locus of I as F varies.

