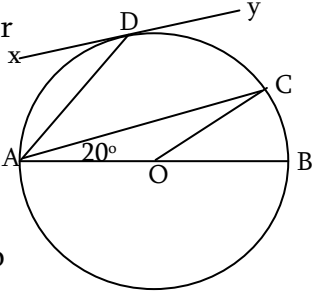


إرشادات عامة:

- يسمح بإستعمال آلة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على المسائل بالترتيب الذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 30
- عدد المسائل: 5

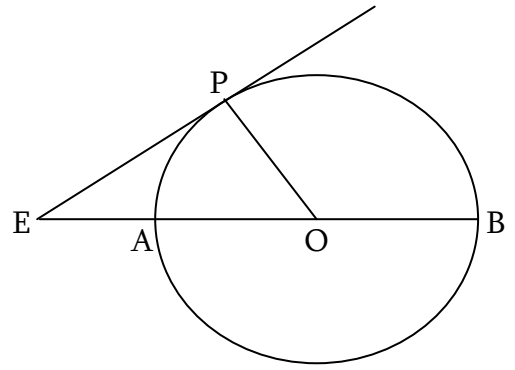
1st exercise: (7½ pts)

In the table below, only one of the proposed answers to each question is correct. Write the number of each question and the corresponding answer, **and justify**.

No.	Questions	Answers		
		A	B	C
1.	Let $A = \frac{4^{n+1} - 4^n}{2^{n+1} + 2^n}$, then a simplified writing of A is:	2^n	0	4^n
2.	Given $E = \sqrt{(7-4\sqrt{3})(7+4\sqrt{3})} + \sqrt{(3-\pi)^2} - \sqrt{(\pi-1)^2}$ then E =	5	$3-2\pi$	- 1
3.	ABC is a triangle such that: $AB = \frac{1}{\sqrt{7+2}} + \frac{1}{\sqrt{7-2}}$, $AC = \frac{14}{3\sqrt{7}}$ and $BC = \frac{28}{3\sqrt{14}}$. Then triangle ABC is:	Equilateral	Isosceles	Right isosceles
4.	Given the polynominal $P(x) = x^2 + 3x - a$. If $x = \sqrt{5} - 1$ is a root of $P(x)$, then a =	$3 + \sqrt{5}$	$9 + 7\sqrt{5}$	$3 + 3\sqrt{5}$
5.	In the figure to the right: <ul style="list-style-type: none"> • Given a circle (C) of center O and diameter [AB]. • C is a point on (C) such that $\angle CAB = 20^\circ$. • D is the midpoin of arc AC, and (xy) is tangent to (C) at D.  <p>The measure of angle ADx =</p>	35°	70°	20°

2nd exercise: (7 pts)

Consider a circle (C) of center O and diameter [AB]. P is a point on the circle (C) and E belongs to the semi-straight-line [OA) as shown in the figure.



Part A:

Given the following lengths:

$$EA = 3\sqrt{2}(\sqrt{3}+1) + (\sqrt{2}-1)(\sqrt{2}-2) - 4$$

$$AB = 8\sqrt{6} - 2\sqrt{3} \times \sqrt{2} + 2\sqrt{150}, \text{ and } EP = 3\sqrt{38}$$

- 1) Write EA in the form of $a\sqrt{b}$ where a and b are two natural integers to be determined. (3/4 pt)
- 2) Show that the radius R of (C) is $8\sqrt{6}$. (3/4 pt)
- 3) Verify that (EP) is tangent to (C) at P. (1pt)

Part B:

Through the point E, another tangent (EQ) is issued to (C). (Q is the point of tangency)

In this part, suppose that:

$$EP = 9ab - 15b + 3a - 5.$$

and

$$EQ = (b+1)^2 + (2b+1)(3b-1) + (b+1)(2b+1). \text{ (where } a \geq 2 \text{ and } b \geq 0).$$

- 1) Write EP in the form of a product of two factors. (1pt)
- 2) Develop EQ and show that EQ is a perfect square. (1 1/2 pts)
- 3) a) What can you say about the two tangents [EP] and [EQ]? (1/2 pts)
b) Calculate b knowing that a = 2. (1 1/2 pts)

3rd exercise: (7 pts)

Given a circle (C) of center O, radius R, and diameter [AB]. (D) is a straight – line tangent to (C) at A, and M is a variable point on (D) such that $AM > AB$. Through the point M, another tangent (ME) to (C) is drawn and cuts (AB) at F (E is the point of tangency). The straight – line (OE) intersects (D) at G and S is the orthogonal projection of O on [FG].

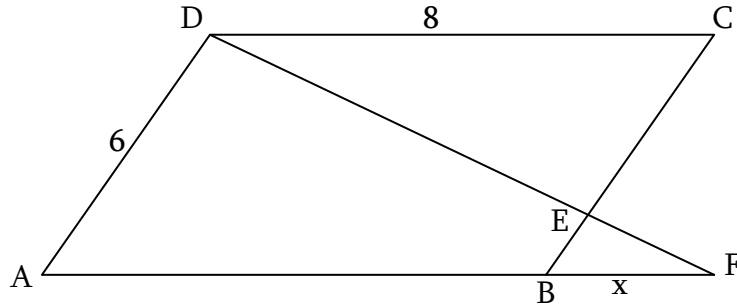
- 1) Draw a figure. (1pt)
- 2) Show that the triangle AME is isosceles, and (MO) is the perpendicular bisector of [AE]. (1 1/2 pts)
- 3) Show that O is the orthocenter of triangle GMF, then deduce that the points M, O, and S are collinear. (1 1/2 pts)
- 4) a) Show that the points O, E, F and S belong to the same circle whose center K is to be determined. (1 1/2 pts)
b) Deduce that $OES = AFG$. (1/2 pt)
- 5) Let I be the midpoint of segment [AE]. Determine the locus of I as M varies on (D). (1pt)

4th exercise: (3 pts)

Consider a parallelogram ABCD and let E be a point on [BC]. The straight – lines (DE) and (AB) interest at F. (see figure below)

1) Show that $\frac{EB}{EC} = \frac{BF}{AB}$. (1½ pts)

2) Suppose that DC = 8cm, AD = 6cm, BE = 2, and BF = x. Calculate x. (1½ pts)



5th exercise: (5½ pts)

The unit of length is the cm and $x > 0$. In the adjacent figure, ABC is a right isosceles triangle at B such that $AB = x + 1$, ABEF is a rectangle of dimensions $EF = x + 1$ and $EB = x$, and (C) is a semi-circle of center B and diameter [EI].

1) Express in terms of x: (1 ½ pts)

- A_1 = the area of the rectangle ABEF.
- A_2 = the area of the triangle ABC.
- A_3 = the area of the semi-circle (C).

2) Let $S(x)$ be the area of the shaded region in the figure.

a) Show that $S(x) = \frac{1}{2}[(3 - \pi)x^2 + 4x + 1]$. (2 pts)

b) Suppose that $S(x) = \frac{1}{2}$. Show that $x = \frac{4}{\pi - 3}$. (2 pts)

