الشهادة المتوسطة   التجربة الثانية لعام 2010 - 2011

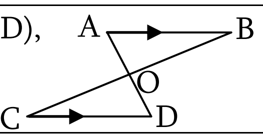
مسابقة في الرياضيات الانكليزي	المدّة : ساعتين	الإسم :	الرقم :

إرشادات عامة:

- يسمح بإستعمال ألة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على ألمسائل بالترتيب الذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 30
- عدد المسائل: 5

1st exercise: (6pts)

In the table below, only one of the proposed answers to each question is correct. Write the number of each question and the corresponding answer and justify.

No.	Questions	Answers a	Answers b	Answers c
1.	Given the polynomial $P(x) = 4x^3 - 4x^2 + x$, then the equation $P(x) = 0$ admits	2 roots as integers	3 roots as integers	1 root as an integer
2.	Given $E = \sqrt{(2-\pi)^2} - \sqrt{(2\pi-3)^2} + \sqrt{(10-3\pi)^2}$ then E =	5	$11 - 4\pi$	$5 + 4\pi$
3.	Given a linear function f such that – 2 has an image 9 by f. The image of 3 by f is:	$\frac{-27}{2}$	$\frac{27}{2}$	$\frac{3}{-2}$
4.	A population of bacteria is formed of 1000 bacteria. Knowing that each hour the population increases by 10%, then after 2 hours the number of bacteria is	2200	1210	1300
5.	In the adjacent figure (AB) // (CD), AB = 6 , CO = 1 and BC = 4 then CD = 	2	3	1

2nd exercise: (6 ½ pts)

Given a triangle LAR such that:

$$LA = \frac{5^{248} - 3^2 \times 25^{123}}{4 \times 125^{82}} \quad , \quad RA = (\sqrt{2}-1)(\sqrt{2}+1) - (2\sqrt{2}-3)^2 - 8(\sqrt{2}-2) \quad \text{and} \quad LR = \frac{4.5 \times 56 \times 1.2}{0.9 \times 4.2 \times 20}$$

(see figure below).

1) a) Show that LR = LA = 4. (2pts)

   b) Write RA in the form $a\sqrt{2}$ where a is a natural integer to be determined. (1pt)

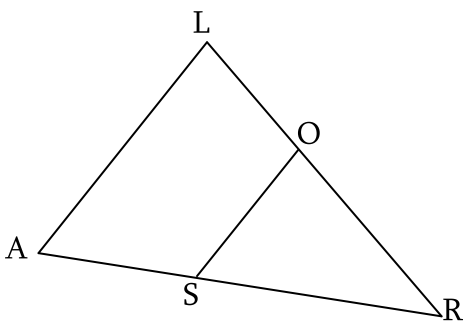
   c) Deduce the nature of triangle LAR. (¾ pt)

2) Let O be the point of [LR] and S be the point of [RA] such that $RO = \frac{2\sqrt{2}-1}{\sqrt{2}+1}$ and $RS = 5\sqrt{2} - 6$.

   a) Rationalize the denominator of RO. (¾ pt)

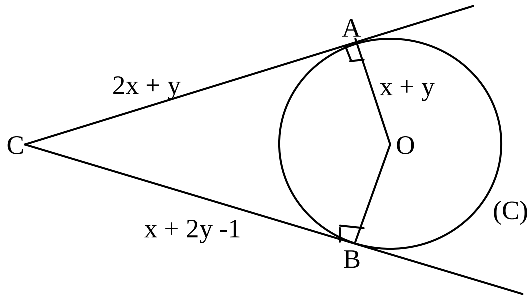
   b) Show that the straight lines (SO) and (LA) are parallel. (1pt)

   c) Calculate SO then write SO in scientific notation. (1pt)



3rd exercise: (4 ½ pts)

In the figure to the right:



- (C) is a circle of center O and radius OA = OB = x + y.
- [CA] and [CB] are two tangents to (C) at A and B such that CA = 2x + y and CB = x + 2 y – 1 (x and y are non zero natural integers)
- The perimeter of the quadrilateral CAOB is 14 cm.
- **<u>Note</u>: Don't draw the figure.**

1) State a relation between CA and CB. Justify. (½ pt)
2) Consider the system $\begin{cases} x - y = -1 \\ x + y = 3 \end{cases}$

   Show that the above system is a modeling or a translation of the given information. (1 ½ pts)
3) a) Solve the system then deduce the lengths CA, CB, and AO. (1 ½ pts)

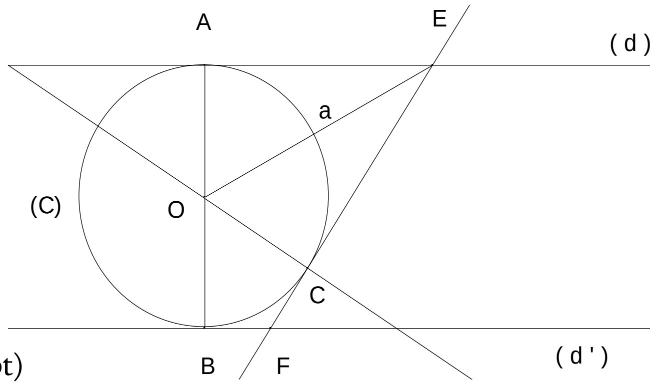
   b) Calculate CO. (1pt)

<u>4th exercise: (3 ½ pts)</u>

In a rectangle the width y and the length x are proportional to the numbers 3 and 4 (x and y are non zero natural numbers in cm)

1) Show that y is a linear function of x. (1pt)
2) Represent in an orthonormal system the obtained linear function. (1pt)
3) Determine graphically and by calculation the length if the width is 6cm. (1 ½ pts)

<u>5th exercise: (9 ½ pts)</u>



Given two fixed parallel straight lines (d) and (d') such that the distance between them is equal to **a** (in cm). (C) is a variable circle of center O and tangent to (d) and (d') at A and B respectively. E is a point of (d) such that OE = **a**. The tangent to (C) through E cuts (C) at C and (d') at F. **(See figure to the right and don't draw the figure).**

1) a) Show that the points O , A and B are collinear. (¾ pt)

   b) Calculate the radius R of circle (C) in terms of **a**. (½ pt)

   c) Can you say that the radius of (C) is proportional to **a**? Justify. (½ pt)

   d) Determine the locus of O as A and B vary. (1pt)
2) a) Show that triangle AOE is semi-equilateral, then deduce AE without using pythagoras' theorem.(1 ½ pts)

   b) Determine how many tangents are issued through each point E and F to (C), then prove that triangle EOF is semi-equilateral. (1pt)

   c) Show that $EF = \frac{2a\sqrt{3}}{3}$. (¾ pt)
3) Let M be the midpoint of [EF]. Calculate OM. (½ pt)
4) a) Justify that the quadrilateral BFEA is a right trapezoid, then prove that (OM) and (AE) are parallel. (1pt)

   b) Let H be the point of intersection of (AB) and (EF). Show that $\frac{AE}{HE} = \frac{OM}{HM}$ (1pt)

5) Calculate $B\hat{O}F$ and $A\hat{H}E$ then deduce that B is the midpoint of [OH]. (1pt)