ليسه دي زار

التجربة الثانية لعام 2009 - 2010   الشهادة المتوسطة

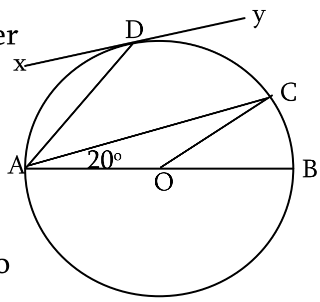
مسابقة في الرياضيات الإنكليزي	المدّة : ساعتين	الإسم :	الرقم :

إرشادات عامة:

- يسمح بإستعمال ألة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على ألمسائل بالترتيب االذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 30
- عدد المسائل: 5

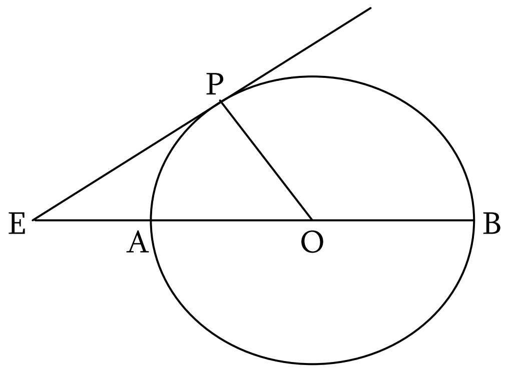
1st exercise: (7½ pts)

In the table below, only one of the proposed answers to each question is correct. Write the number of each question and the corresponding answer, **and justify**.

No.	Questions	Answers		
		A	B	C
1.	Let $A=\dfrac{4^{n+1}-4^{n}}{2^{n+1}+2^{n}}$, then a simplified writing of A is:	2^{n}	0	4^{n}
2.	Given $E=\sqrt{\left(7-4\sqrt{3}\right)\left(7+4\sqrt{3}\right)}+\sqrt{\left(3-\pi\right)^{2}}-\sqrt{\left(\pi-1\right)^{2}}$ then E =	5	$3-2\pi$	- 1
3.	ABC is a triangle such that: $AB=\dfrac{1}{\sqrt{7}+2}+\dfrac{1}{\sqrt{7}-2}$, $AC=\dfrac{14}{3\sqrt{7}}$ and $BC=\dfrac{28}{3\sqrt{14}}$. Then triangle ABC is:	Equilateral	Isosceles	Right isosceles
4.	Given the polynominal $P(x)=x^{2}+3x-a$. If $x=\sqrt{5}-1$ is a root of P(x), then a =	$3+\sqrt{5}$	$9+7\sqrt{5}$	$3+3\sqrt{5}$
5.	In the figure to the right: • Given a circle (C) of center O and diameter [AB]. • C is a point on (C) such that $\widehat{CAB}=20^{\circ}$. • D is the midpoing of arc $\overset{\frown}{AC}$, and (xy) is tangent to (C) at D.  The measure of angle $\widehat{ADx}$ =	35°	70°	20°

2nd exercise: (7 pts)

Consider a circle (C) of center O and diameter [AB]. P is a point on the circle (C) and E belongs to the semi-straight-line [OA) as shown in the figure.



Part A:

Given the following lengths:

$EA = 3\sqrt{2}\left(\sqrt{3}+1\right)+\left(\sqrt{2}-1\right)\left(\sqrt{2}-2\right)-4$

$AB = 8\sqrt{6}-2\sqrt{3}\times\sqrt{2}+2\sqrt{150}$, and $EP = 3\sqrt{38}$

1) Write EA in the form of $a\sqrt{b}$ where a and b are two natural integers to be determined. (¾ pt)
2) Show that the radius R of (C) is $8\sqrt{6}$. (¾ pt)
3) Verify that (EP) is tangent to (C) at P. (1pt)

Part B:

Through the point E, another tangent (EQ) is issued to (C). (Q is the point of tangency)

In this part, suppose that:

EP = 9ab – 15b + 3a - 5.

and

EQ = $(b + 1)^2 + (2b + 1)(3b – 1) + (b + 1)(2b + 1)$. (where $a \geq 2$ and $b \geq 0$).

1) Write EP in the form of a product of two factors. (1pt)
2) Develop EQ and show that EQ is a perfect square. (1 ½ pts)
3) a)What can you say about the two tangents [EP] and [EQ] ? (½ pts)

   b) Calculate b knowing that a = 2. (1½ pts)

3rd exercise: (7 pts)

Given a circle (C) of center O, radius R, and diameter [AB]. (D) is a straight – line tangent to (C) at A, and M is a variable point on (D) such that AM > AB . Through the point M, another tangent (ME) to (C) is drawn and cuts (AB) at F (E is the point of tangency). The straight – line (OE) intersects (D) at G and S is the orthogonal projection of O on [FG].

1) Draw a figure. (1pt)
2) Show that the triangle AME is isosceles, and (MO) is the perpendicular bisector of [AE]. (1 ½ pts)
3) Show that O is the orthocenter of triangle GMF, then deduce that the points M, O, and S are collinear. (1 ½ pts)
4) a) Show that the points O, E, F and S belong to the same circle whose center K is to be determined.(1½ pts)

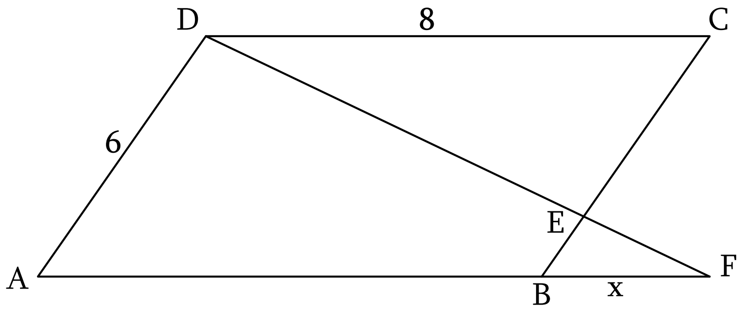
   b) Deduce that $\widehat{OES} = \widehat{AFG}$. (½ pt)
5) Let I be the midpoint of segment [AE]. Determine the locus of I as M varies on (D).(1pt)

4th exercise: (3 pts)

Consider a parallelogram ABCD and let E be a point on [BC]. The straight – lines (DE) and (AB) interest at F. (see figure below)

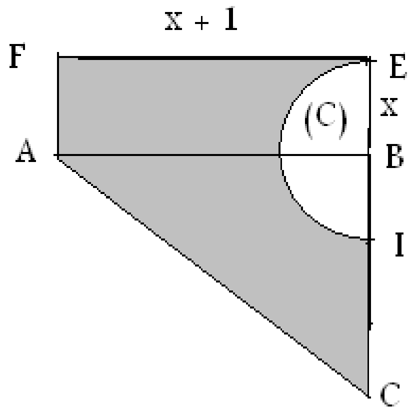
1) Show that $\frac{EB}{EC} = \frac{BF}{AB}$. (1½ pts)

2) Suppose that DC = 8cm, AD = 6cm, BE =2, and BF = x. Calculate x. (1½ pts)



5th exercise: (5½ pts)

The unit of length is the cm and x > 0. In the adjacent figure, ABC is a right isosceles triangle at B such that AB = x + 1, ABEF is a rectangle of dimensions EF = x + 1 and EB = x, and (C) is a semi-circle of center B and diameter [EI].



1) Express in terms of x: (1 ½ pts)
   - A_1 = the area of the rectangle ABEF.
   - A_2 = the area of the triangle ABC.
   - A_3 = the area of the semi-circle (C).
2) Let S(x) be the area of the shaded region in the figure.

a) Show that $S(x) = \frac{1}{2}\left[(3-\pi)x^2 + 4x + 1\right]$. (2 pts)

b) Suppose that $S(x) = \frac{1}{2}$. Show that $x = \frac{4}{\pi - 3}$. (2 pts)