| مسابقة في الرياضيات الانكليزي |
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إ رشادات عامة:
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## $1^{\text {st }}$ exercise: ( $71 / 2$ pts)

## In this exercise, the four parts are independent.

1) Consider the following equation (E): $\frac{4^{4}-2^{3}}{2^{3}}=2^{2 n+1}-1$.

Calculate the value of the unknown n. ( $11 / 2 \mathrm{pts}$ )
2) In the plane referred to an orthonormal system of axes x'ox, y'oy, consider the points $A(1,-1)$, $B(3,3)$ and $C(1,7)$.

## We admit without proof that triangle $A B C$ is isosceles at $B$.

Determine an equation of the internal bisector of angle $A \hat{B} C$. ( $1^{1 / 2} \mathrm{pts}$ )
3) Consider a rectangle of perimeter 20 m and length $\ell$.
a) Express the area A in terms of $\ell$. Can you say that A is a linear function of $\ell$ ? Explain. ( $11 / 2 \mathrm{pts}$ )
b) Show that $A=25-(\ell-5)^{2} \cdot(1 / 2 \mathrm{pt})$
c) Deduce the value of $\underline{\ell}$ for which the area is maximum. (1pt)
4) The area of a room is $20 \mathrm{~m}^{2}$. The decorator offered a $5 \%$ discount for the tiling (تبليط) so that the total price of tiling for the room after discount is 4275\$. Calculate the price of each $m^{2}$ of tiling before the discount. ( $1^{1 / 2} \mathrm{pts}$ )

## $2^{\text {nd }}$ exercise: ( $41 / 2 \mathrm{pts}$ )

$A B C D$ is a rectangle such that $A B=6 \mathrm{~cm}$ and $B C=4 \mathrm{~cm}$.
M and N are two points of $[\mathrm{AB}]$ and $[\mathrm{CD}]$ respectively such that $A M=C N=x$.

1) Write an encirclement for $\underline{x}$. $(1 / 2 \mathrm{pt})$
2) a) Calculate the area $A_{1}$ of triangle CBM then deduce the area $A_{2}$ of the parallelogram AMCN. ( $11 / 2 \mathrm{pts}$ )
b) What type of functions are $A_{1}$ and $A_{2}$ ? Justify. (1pt)
3) Determine $\underline{x}$ so that the area of the parallelogram AMCN is the double
 that of the shaded part. ( $11 / 2 \mathrm{pts}$ )

## $3^{\text {3rd }}$ exercise: ( 12 pts )

In an orthonormal system x'ox, y'oy, consider the points $A(2,-2), B(-1,2)$ and $C(6,1) . M\left(\frac{5}{2}, m\right)$ is a variable point where $m$ is a real parameter and (d) is the straight-line of equation $y=\frac{3}{4} x-\frac{7}{2}$.

1) Plot the points A, B and C. (1pt)
2) Show that (d) passes through the points $A$ and $C$ then draw (d). ( $11 / 4 \mathrm{pts}$ )
3) a) Determine the set of points M as m varies. ( 1 pt )
b) Calculate $a_{(B C)}$, the slope of $(\mathrm{BC})$ and verify that $a_{(A M)}=2 m+4$. (11/2pts)
c) Determine m so that the two straight-lines ( BC ) and ( AM ) are perpendicular. (1pt)

In what follows, take $m=\frac{3}{2}$ so that $M\left(\frac{5}{2}, \frac{3}{2}\right)$
4) a) Show that $M$ is the midpoint of $[B C] \cdot(3 / 4 \mathrm{pt})$
b) Calculate the lengths BC and AM then verify that $A M=\frac{B C}{2} \cdot\left(1^{1 / 2} \mathrm{pts}\right)$
c) Deduce the nature of triangle ABC. (1pt)
5) Let $(\Delta)$ be the straight-line perpendicular to (AC) and passing through the point $K(3,4)$.
a) Determine the equation of $(\Delta)$. (1pt)
b) Verify that the point $S(9,-4)$ is the point of intersection between $(\Delta)$ and the straight-line (L) having an ordinate of the origin 2 and an x -intercept 3. (2pts)

## $4^{\text {th }}$ exercise: ( $81 / 2 p$ ts)

The manager of a theatre proposes two options to his spectators:
Option 1: Pay a sum of $60 €$ per session.
Option 2: Pay an annual subscription of $210 €$ and obtain a discount of $25 \%$ on each session that costs $60 €$.

1) What is the advantageous option if the spectator wants to attend 15 sessions in one year? Justify.( $\left.1^{1 / 2} 2 p t s\right)$
2) Designate by $x$ the number of sessions attended in one year.

- $f(x)$ is the sum paid in $€$ in option 1.
- $g(x)$ is the sum paid in $€$ in option 2 .
a) Express $f(x)$ in terms of $x$ then show that $g(x)=45 x+210 .\left(1 \frac{1}{2} p t s\right)$
b) Complete the following table and show your work: ( $11 / 2 \mathrm{pts}$ )

| $X$ | 0 | 2 | 14 |
| :--- | :--- | :--- | :--- |
| $f(x)$ |  |  |  |
| $g(x)$ |  |  |  |

c) Represent graphically the two straight-lines $\left(d_{1}\right)$ and $\left(d_{2}\right)$ representing the two functions $f$ and $g$ respectively.
Note: the plane is referred to an orthogonal system where:

- On the x -axis, 2 cm corresponds to 1 session.
- On the y-axis, 1 cm corresponds to $50 €$. ( $1 \frac{1}{2} \mathrm{pts}$ )

3) a) Solve the equation $f(x)=g(x)$ and interpret the obtained result. (1pt)
b) Discuss graphically, according to the values of $x$, the most advantageous option for the spectator. ( $1^{1 / 2}$ pts)

## $5^{\text {th }}$ exercise: ( $71 / 2 \mathrm{pts}$ )

Consider the circle (C) of center $O$ and diameter $A C=10 \mathrm{~cm}$. D is the point of (C) such that $\overparen{C} D=60^{\circ}$. Let I be the orthogonal projection of D on [AC]. (DI) cuts the circle in E and the two straight-lines ( AD ) and (EC) meet at F.

1) Draw a figure in true measures. (1pt)
2) Show that (DO) and (FC) are parallel then deduce that D is the midpoint of $[\mathrm{AF}]$. ( $1 \frac{1}{2} \mathrm{pts}$ )
3) Calculate the lengths FC, FA and AI. (2pts)
4) In this part, consider that $D$ is a variable point on ( $C$ ), and $M$ is the midpoint of [AD].
a) What does (OM) represent for [AD]? Justify. (1pt)
b) Deduce the locus of M as D varies on (C). ( 1 pt )
c) Let G be the centroid of triangle ADC. Determine the locus of G. (1pt)
