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| مسابقة في الرياضيات الإنكليزي | المدة : ساعتان | الإسم : | الرقم : |
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إرشادات عامة:

- يسمح باستعمال آلة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على المسائل بالترتيب الذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 30

1st exercise: (6½ pts)

In the following table, only one of the proposed answers to each question is correct. Write the number of each question and the corresponding answer, and **justify**.

| No. | Questions | Answers | | |
|-----|--|----------------------|-------------------|----------------------------|
| | | A | B | C |
| 1. | a is a real number such that $a = \frac{3\sqrt{5} - 5}{2}$. Then the numerical value of $a(a+5)$ is (1 pt) | 5 | 10 | $\frac{9\sqrt{5} - 25}{4}$ |
| 2. | Given the straight lines: $(d): y = (\sqrt{3} - 5)x - 1$ and $(d'): 22y - (\sqrt{3} + 5)x + 2 = 0$. Then (d) & (d') are..... (1 pt) | Parallel | Perpendicular | Confounded |
| 3. | If $x = -3\sqrt{2}$ is a root of $P(x) = x^2 + a\sqrt{2}x + 12$, then $a = \dots$ (1pt) | $\frac{6}{\sqrt{2}}$ | $2 + 1.5\sqrt{2}$ | 5 |
| 4. | If $\sqrt{x-8} + 7 = 8$, then $\sqrt{x} = \dots$ (1pt) | $1 + 2\sqrt{2}$ | $\sqrt{2}$ | 3 |
| 5. | The price of an item is 350\$. After sales, the new price becomes 280\$. The percentage of reduction is..... (1pt) | 20% | 21% | 15% |
| 6. | The point $A\left(2^{-1} - \frac{3}{5}; (\sqrt{2} - 1)^2 + 2\sqrt{2}\right)$ belongs to the straight-line of equation: (1½ pts) | $y = x + 4$ | $y = 10x + 2$ | $y = 10x + 4$ |

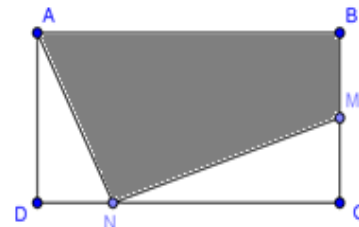
2nd exercise: (7½ pts)

Consider in the orthonormal system of axes $(x'Ox \text{ \& } y'Oy)$, the straight lines $(d): 2y + 6 = x$ and $(d'): y + 2x = 5$ and the points $B(4; -1)$ & $E(4; -3)$. (The unit of length is the cm)

- 1) Determine the relative position of (d) and (d') . (1pt)
- 2) Draw the two straight lines (d) and (d') . (1 pt)
- 3) Show that E belongs to (d') and B belongs to (d) then place the points E and B . (1 pt)
- 4) (d) cuts $x'Ox$ in D and (d') cuts $y'Oy$ in L . Calculate the coordinates of D and L . (1 pt)
- 5) Designate by I the point of intersection of (d) and (d') . Calculate the coordinates of I . (¾ pt)
- 6) a) Calculate the length of $[LD]$. (½ pt)
b) Show that the points O, D, I and L belong to the same circle (C) whose center S and radius are to be determined. (1¼ pts)
- 7) Show that the tangent (T) to the circle (C) at L has an equation $5y - 6x - 25 = 0$. (1 pt)

3rd exercise: (9 pts)

- ✓ $ABCD$ is a rectangle such that : $AB = 4\text{cm}$ and $AD = 2\text{cm}$.
 ✓ M & N are any two points on $[BC]$ & $[CD]$ respectively such that $DN = BM = x$, where $0 < x < 2$. (DO NOT DRAW)
- 1) Determine $A_1(x)$, the area of triangle MCN as a function of x . (¾ pt)
 - 2) $A_2(x)$ is the area of triangle DNA .
 - a) Calculate $A_2(x)$ as a function of x . (½pt)
 - b) Write the expanded form of $P(x)$ where $P(x) = A_1(x) + A_2(x)$.



What does $P(x)$ represent?(¾pt)

- 3) a) Prove that the area of the shaded region is given by: $E(x) = \frac{-x^2 + 4x + 8}{2}$. (1pt)
 - b) Develop: $Q(x) = (x-1)(x-3)$ (¼ pt)
 - c) Calculate the value x if $E(x) = 5.5\text{cm}^2$. (1 pt)
- 4) a) Prove that the area of ANM is: $A = \frac{8-x^2}{2}$. (1pt)
 - b) Calculate the value x if the area of the triangle ANM is 3.28cm^2 . (1 pt)
- 5) The two straight-lines (MN) and (AD) intersect in I .
 - a) Show that $DI = \frac{2x-x^2}{4-x}$. (1 pt)
 - b) Calculate the numbers a and b so that $3x^2 - 7x + 4 = (x-1)(ax+b)$. (¾ pt)
 - c) Deduce x when $DI = \frac{1}{3}$. (1 pt)

4th exercise: (7 pts)

Consider The circle (C) of center O and diameter $[AB]$. (d) , the perpendicular bisector of $[OA]$ cuts $[OA]$ at H and the circle (C) in E . Let K be the symmetric of the point O with respect to E .

- 1) Draw a figure. (½ pt)
- 2) Show that (KA) is tangent to (C) at A using two methods:
 - a. Converse of Thales' Theorem. (1 pt)
 - b. Right Triangle. (1 pt)
- 3) Let $OA = x$ and $KA = 3\sqrt{3}\text{cm}$; calculate the radius of (C) then deduce the nature of triangle AEK . (1 ½ pts)
- 4) Let M be the point on (C) such that $BM = 3\text{cm}$. (**M and K are on the same side with respect to the diameter**)
 Use suitable equilateral triangles; show that the quadrilateral $AEMB$ is an isosceles trapezoid. (1½pts)
- 5) In this part, M is a variable point on the circle (C) of radius 3cm and designate by G the center of gravity of triangle MAB . Calculate OG then deduce the locus of G as M describes (C) . (1½ pts)

