مسابقة في الرياضيات الإنكليزي

## $1^{\text {st }}$ exercise: ( 7 pts )

In the following table, only one of the proposed answers to each question is correct. Write the number of each question and the corresponding answer, and justify.

| № | Questions | Answers |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | a | b | c |
| 1. | The two straight lines (d) and (D) of respective equations: $y=\left(\frac{\frac{3}{7}-\frac{1}{8}}{\frac{1}{8}-\frac{3}{7}}+3\right) x+2$ and $x=\frac{1}{2} y-1$ are $\qquad$ (11/2pts) | Parallel | Perpendicular | Confounded |
| 2. | In the figure below, we have: <br> * ABC is a right triangle at A. <br> * PNMA is a rectangle. <br> ${ }^{*} A M=x \mathrm{~cm} ;(x>0)$ $\begin{aligned} & * A C=\sqrt{6-2 \sqrt{5}} \times \sqrt{6+2 \sqrt{5}}-\sqrt{(2-\sqrt{3})^{2}}-\sqrt{3} \\ & { }^{*} A B=(1-\sqrt{2})^{2}+\sqrt{8} \mathrm{~cm} \end{aligned}$ <br> Then $A P=\ldots$ <br> ( $2^{1 ⁄ 2}$ pts) | $\frac{3}{2} x$ | $3-1.5 x$ | $\frac{6}{x}$ |
| 3. | If $\left\{\begin{array}{l}3 \sqrt{x}-2 \sqrt{y}=4 \\ 3 \sqrt{x}+2 \sqrt{y}=8\end{array}\right.$ then $\sqrt{x^{3}-60 y^{3}}$ is ... ( $11 / 2 \mathrm{pts}$ ) | 2 | 1 | 0 |
| 4. | In a store, the price of an electrical gadget is reduced by $20 \%$ during the season of holidays, then after this period the price has been raised by $25 \%$, then..... <br> (11⁄2pts) | Then the final price is less than the initial price | Then the final price is greater than the initial price | Then the final price is equal the initial price |

$2^{\text {nd }}$ exercise: ( $61 / 4 \mathrm{pts}$ )
In the following figure, we have:

* $(\mathrm{BC})$ is parallel to (DE).
* $\mathrm{AE}=3 \mathrm{~cm}$ and $\mathrm{EC}=6 \mathrm{~cm}$.
* $\mathrm{DE}=\boldsymbol{x}+2 \boldsymbol{y}-2$ and $\mathrm{BC}=2 \boldsymbol{x}+3 \boldsymbol{y}-6$
( $x$ and $y$ are two real numbers for which DE and BC exist) NOTE: DO NOT DRAW THE FIGURE.

1) Show that: $\boldsymbol{y}=-\frac{1}{3} x$. ( $1 \frac{1}{2}$ pts)

2) Let $\boldsymbol{g}$ be a function such that $\boldsymbol{g}(\boldsymbol{x})=\boldsymbol{y}$.
a) What is the nature of $g$ ? Justify. $(1 / 2 p t)$
b) What is the sense of variation of $g$ ? Justify. ( $1 / 2 \mathrm{pt}$ )
c) Represent $\boldsymbol{g}$, graphically by a straight line (d), in an orthonormal system of axes (x'Ox, y'Oy). ( $3 / 4 \mathrm{pt}$ )
d) $f$ is a decreasing affine function of representative straight line (d'). How would you choose the director coefficient of $f$ so that ( $d^{\prime}$ ) is closer to the ordinate axis than (d). (1pt)
3) Calculate the perimeter of DECB, when the two conditions below are satisfied:

* (d) passes through a point of abscissa $\boldsymbol{x}=9$.
* The perimeter of ADE equals to 6 cm . (2pts)


## $3^{\text {rd }}$ exercise: ( $101 / 4 \mathrm{pts}$ )

In an orthonormal system $(\boldsymbol{x} \cdot \boldsymbol{O} \boldsymbol{x}, \boldsymbol{y} \cdot \boldsymbol{O} \boldsymbol{y})(1 \mathrm{unit}=1 \mathrm{~cm})$, given the points $A\left(\frac{\boldsymbol{m}}{2} ; \boldsymbol{n}\right), B(-1-\boldsymbol{m} ; 4-\boldsymbol{n})$, $C(4 ;-3)$ and $D(2 ;-6) .(\mathrm{d})$ is the straight line of equation $\frac{1}{2} y+2 x-2=-1$.

1) a) Show that the reduced form of the equation of (d) is $y=-4 x+2$. $(1 / 2 \mathrm{pt})$
b) Deduce the coordinates of I and J, the intersection points between (d) and the axes of system: $\boldsymbol{x} \cdot \boldsymbol{O} \boldsymbol{x}$ and $\boldsymbol{y}$ ' $\boldsymbol{O} \boldsymbol{y}$ respectively. (1pt)
c) Verify, by calculation, that the point D belongs to (d). ( $1 / 2 \mathrm{pt}$ )
d) Draw (d) in the orthonormal system of axes. ( $1 / 2 \mathrm{pt}$ )
2) Let $P$ be the symmetric of $D$ with respect to the ordinate axis $y$ ' $\mathrm{O} y$ and $Q$ the symmetric of $P$ with respect to $x^{\prime} \mathrm{Ox}$.
a) Determine the coordinates of the two points $P$ and Q . ( 1 pt )
b) Determine the slope of the straight line ( PQ ) and deduce its equation. (1pt)
$3)(\Delta)$ is the straight line of equation $\boldsymbol{y}=\boldsymbol{x}+3$.
a) Show that if $(\Delta)$ passes through $A$ and $B$ then $m$ and $n$ verify the system: $\left\{\begin{array}{l}2 \boldsymbol{n}-\boldsymbol{m}=6 \\ n-m=2\end{array}\right.$. $(3 / 4 \mathrm{pt})$
b) Deduce then the coordinates of A and B when they belong to $(\Delta)$ and then plot them. ( $11 / 4 \mathrm{pts}$ )

In what follows, suppose that. $\boldsymbol{m}=2, \boldsymbol{n}=4$ and $\mathrm{CA}=\sqrt{58}$.
4) a) Show that $B C=\sqrt{58} .(1 / 2 \mathrm{pt})$
b) Let $K$ be the midpoint of $[\mathrm{AB}]$ and let (C) be the circle circumscribed about the triangle $K B C$. Determine the coordinates of S , the center of (C), and calculate its radius. ( $11 / 4 \mathrm{pts}$ )
c) Precise the position of the point $F(0 ;-0.5)$ with respect to the circle (C). (Interior or exterior to (C)) ( $3 / 4 \mathrm{pt}$ )

6 ) Find the equation of the tangent ( T ) to the circle ( C ) at $B$. ( $1 \frac{1}{4} \mathrm{pts}$ )
$4^{\text {th }}$ exercise: ( $61 / 2 \mathrm{pts}$ )
$(\mathrm{S})$ is the circle of center $O$ and of diameter $[A B]$ such that $A B=6 \mathrm{~cm}$. Let $M$ be the midpoint of $[O B]$ and $C$ be a point on $(\mathrm{S})$ such that $B C=3.6 \mathrm{~cm}$.

1) Draw a neat and clear figure that will be completed progressively. ( $3 / 4 \mathrm{pt}$ )
2) Determine the nature of the triangle $A B C$, and then deduce that $A C=4.8 \mathrm{~cm}$. ( $1 \frac{1}{4} \mathrm{pts}$ )
$3)>$ The straight line parallel to $(A C)$ through $O$ and the tangent to $(S)$ at $C$ intersect at $D$. $>(O D)$ cuts $(B C)$ at $R$.
a) Show that $[O R)$ is the bisector of $C \hat{O} B .(1 \mathrm{pt})$
b) Prove that the triangles $C O D$ and $O B D$ are congruent. $(3 / 4 \mathrm{pt})$
c) Deduce that $(B D)$ is tangent to $(\mathrm{S})$ at $\boldsymbol{B}$. $(3 / 4 \mathrm{pt})$
d) Verify that the points $O, C, D$ and $B$ belong to the same circle whose diameter is to be determined. (1pt)
3) Suppose in this part that the point $C$ varies on the circle (S). Designate by $G$ the centroid of the triangle ABC . Determine the line on which the variable point G varies as C describes the circle ( S ). ( 1 pt )
