مسابقة في الرياضيات الفرنسي	المدّة : ساعتان	الإسم :	الرقم :

إرشادات عامة:

- يسمح بإستعمال آلة حاسبة غير قابلة للبرمجة.
- يمكن الإجابة على المسائل بالترتيب االذي تريد.
- يرجى الإجابة بخط واضح ومرتب.
- العلامة القصوى من 30

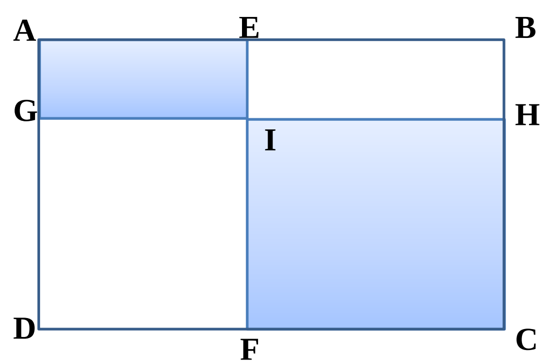
1st exercise: (11pts)

The parts of this exercise are independent:

1) Consider the circle $C(O;6cm)$. Let M be any point outside (C) so that $MO = x\,cm$. The tangent to (C) issued from M cuts (C) at E where $ME = y\,cm$.
   a. Draw a sketch. **(½pt)**
   b. Use a suitable right triangle, prove that $x^2 - y^2 = 36$. **(¾pt)**
   c. Calculate the exact values of x & y knowing that the perimeter of triangle MOE is $24cm$. **(1½pts)**

2) Consider the system: $\begin{cases} x + y = 180 \\ 6x + 15y = 1800 \end{cases}$
   a. Solve the above system. **(1pt)**
   b. **Application:** The total price of two articles is 180$. When the price x of the 1st-item **is reduced by 40%** and the price y of the 2nd-item **is increased by 50%**, then the total price of both items **remains the same.**
      i. Model the above text into a system of first degree equations in two unknowns. **(1½pts)**
      ii. Deduce the initial price of each item. **(¾pt)**

3) **Consider the polynomial:** $E(x) = 4(x+1)^2 + (8x+8)(x-5)$
   a. Factorize $E(x)$, then deduce its roots. **(1½pts)**
   b. **In the adjacent figure,** (The unit of length is cm)
   $ABCD$, $AEIG$ & $IHCF$ are rectangles, so that :
   $IH = 2AE = 8(x+1)$, $AG = x+1$ and
   $$BC = \frac{(7+4\sqrt{2})(7-4\sqrt{2})+16}{\sqrt{5^2-4^2}} - 5.$$
   $(-1 < x < 5)$. *(EF) // (AD) and (GH) // (CD).*



Designate by $\mathcal{A}_1$ and $\mathcal{A}_2$ the respective areas of the rectangles AEIG and IHCF.

i. Prove that $BC = 6cm$. **(1pt)**
ii. Show that $\mathcal{A}_1 = 4(x+1)^2$ and $\mathcal{A}_2 = 8(5-x)(x+1)$. **(1½pts)**
iii. Calculate the value of x so that, the rectangles $IHCF$ & $AEIG$ **have equal areas. (1pt)**

2nd exercise: (12½pts)

Consider in the orthonormal system of axes $x'Ox$ & $y'Oy$ the points $A(3;\ 0)$ & $B(-1;\ 2)$ and the straight lines $(d): 2x-y+4=0$ and $(l): y=mx+3x-1$. **(The unit of length is cm)**

1) a. Determine the slope of (d). **(¼pt)**
   b. Find the value of m so that (d)&(l) are parallel straight-lines. **(¾pt)**
   c. What is the relative position of (d)&(l) when $m=-\frac{1}{2}$? Justify. **(¾pt)**
2) Justify that the point A **belongs to one of the coordinate axes,** then draw the orthonormal system and place A & B .**(¾pt)**
3) **The straight line (d) cuts abscissa axis in E and the ordinate axis in F .**
   a. Determine the coordinates of E & F , then draw (d). **(¾pt)**
   b. Prove that $AO=3cm$ & $AE=5cm$. **(¾pt)**
   c. Verify that B is the midpoint of $[EF]$.**(¾pt)**
4) The straight line joining the points A & B represents an affine function.
   a. Determine the equation of (AB). **(¾pt)**
   b. Deduce that (AB) is the perpendicular bisector of $[EF]$. **(¾pt)**
   c. Calculate the length of $[FE]$, then deduce the nature of the triangle FEA .**(1¼pts)**
   d. Prove that the perimeter of triangle BEA is $(10+2\sqrt{5})cm$. **(1pt)**
5) **(n) is the straight line that is passing through the origin O and perpendicular to (AB)**
   a. Determine the equation of (n). **(¾pt)**
   b. Calculate the coordinates of D the point of intersection of (n)&(AB). **(¾pt)**
   c. **Without calculating** the lengths AD & AB , prove that $\frac{AD}{AB}=\frac{3}{5}$. **(¾pt)**
   d. Deduce the perimeter of triangle DOA .**(¾pt)**
6) (C) is the circle of diameter $[AF]$ and center G.
   a. Prove that (BG) is parallel to (AE). **(½pt)**
   b. **Find without calculating the coordinates of G** the equation of (BG). **(½pt)**

3rd exercise: (6½pts)

Consider a right isosceles triangle AOB of hypotenuse [AB], so that AB=5cm, and K is the midpoint of [AB]. (C) is the circle of center O and radius OA.

1) Draw a clear figure.**(½pt)**
2) Prove that (C) passes through B.**(¾pt)**
3) Calculate the area of (C).**(¾pt)**
4) Let F be diametrically opposite to A, and I be a point of [AB] so that AI=1cm.
   a. Plot H the orthogonal projection of I on (AF).**(¼pt)**
   b. Verify that (IH) is parallel to (OB).**(¾pt)**
5) Calculate the length of [AH]and deduce IH.**(1½pts)**
6) a. Prove that the points F, H, I & B belong to the same circle (C'), whose diameter is to be determined.**(1pt)**
   b. Calculate the diameter IF. **(1pt)**