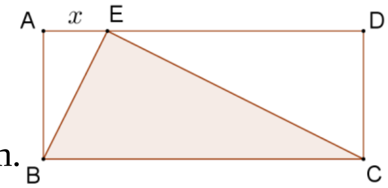
مسابقة في الرياضيات الانكليزي	المدّة : ساعتان	الإسم :	الرقم :

إرشادات عامة:

- يسمح بإستعمال آلة حاسبة غير قابلة للبرمجة.
- يمكن الإجابة على المسائل بالترتيب االذي تريد.
- يرجى الإجابة بخط واضح ومرتب.
- العلامة القصوى من 40.

1st exercise: (10½pts)



Consider a rectangle $ABCD$ of dimensions: $a=\frac{4^7-8^4}{4^6-4^5}$ and $b=\frac{7}{3+\sqrt{2}}-\sqrt{\left(3-2\sqrt{2}\right)^2}$, where AD > AB and the unit of length is the cm.

let E be a point on side $[AD]$ of the rectangle $ABCD$ such that $AE=x$ and $0<x<2$.

1) a) Express $a\ \&\ b$ in the simplest form possible. **(2pts)**
   b) Prove that $AD = 4cm$ and $AB = \sqrt{2}cm$. **(½pt)**
   c) Determine in terms of x the areas $P\ \&\ Q$ of the triangles $AEB\ \&\ DEC$ respectively. **(1½pts)**
2) Justify that area $Q=\frac{(4-x)\sqrt{2}}{2}$ **doesn't** represent a linear function, and deduce its sense of variation. **(1pt)**
3) **Deduce** that the area of the triangle BEC is $2\sqrt{2}cm^2$. **(¾pt)**
4) **Consider the expression:** $J(x)=(x-2)^2-2$.
   a) Write $J(x)$ in the expanded form. **(¾pt)**
   b) Find the roots of $J(x)$. **(½pt)**
   c) Prove that $BE=\sqrt{x^2+2}cm$. **(¾pt)**
   d) Deduce the value(s) of x for which the triangle BEC right at E. **(1¼pts)**
5) **Suppose in this part that** $x=2-\sqrt{2}$ **and the lines** $(EC)\ \&\ (AB)$ **intersect at** F **.**
   Prove that $FA=3\sqrt{2}-4$. **(1½pts)**

2nd exercise: (6pts)

Consider the system of equations: $\begin{cases}2x+y=20\\22x+9y=204\end{cases}$

1) Solve the above system. **(1pt)**
2) Determine the **values of** m so that the couple $(6;\ 8)$ verifies the equation $(m+2)^2x-3my=48$. **(1½pts)**
3) **Application**: Let the perimeter of an **isosceles triangle** ABC with main vertex A be $20cm$. If the lengths of the sides [AB] and [AC] are **increased each** by 10% and the the length of the base $[BC]$ is **decreased** by 10% then the perimeter becomes $20.4cm$
   a) Translate the above text into a linear system of two equations in two unknowns. **(2½pts)**
   b) Deduce the length of each of the sides of the triangle ABC. **(1pt)**

<u>3rdexercise:</u> (8pts)

The parts of this exercise are independent:

1) Given the following table:

$B=\left(\frac{a^2-1}{a^2+1}\right)^2+\left(\frac{2a}{a^2+1}\right)^2$	$D=\left(\frac{3}{5}-\frac{5}{2}\times\frac{-1}{10}\right)\div\left(1-\frac{23}{40}\right)$
$A=\frac{5^{248}-3^2\times 25^{123}}{4\times 125^{82}}-2$	$C=\frac{7^3\times 5^{-4}}{49\times 5^{-6}\times 7}-21$

a) Show that A is a multiple of 2. **(1pt)**

b) Prove that B=1. **(1pt)**

c) Show that $\frac{C}{D}=\frac{1}{2}$. **(2pts)**

d) Prove that the adjacent table is a proportionality table. **(1pt)**

2) Given the following system: $\begin{cases}3(x+1)^2+2(y-3)^2=11\\5(x+1)^2-3(y-3)^2=-7\end{cases}$

a) Solve the above system. **(1½pts)**

b) If $x<-1$ & $y>4$, then show that $e=\frac{\frac{1}{x}+y}{(x+y)^2}$ is a decimal fraction. **(1½pts)**

<u>4rthexercise:</u> (9pts)

In an orthonormal system of axes $x'Ox$ & $y'Oy$, consider the straight line $(D): y=\frac{1}{2}x+1$ and the points $B(2;0)$ and $F\left(\frac{22}{5};\frac{16}{5}\right)$ and the unit of length is the cm.

1) a) Draw the line (D) and plot the point B. **(1pt)**

   b) Verify that F belongs to (D). **(¾pt)**

2) (D) cuts $x'Ox$ in E, and $y'Oy$ in C.

   a) Determine **graphically** the coordinates of E & C. **(½pt)**

   b) Calculate the length EC. **(½pt)**

3) If A is a point of coordinates $A(0;2\sqrt{3})$ then

   a) Prove that $O(0;0)$ is the midpoint of $[EB]$. **(¾pt)**

   b) Deduce that the triangle AEB is equilateral. **(1pt)**

4) **Let (C) be the circle of center B and radius $[AB]$. (C) cuts $[Oy')$ in point R.**

   a) Determine the nature of the quadrilateral $ABRE$. Justify. **(1pt)**

   b) What is the relative position of the point F with respect to the circle (C)? Justify. **(¾pt)**

5) (D) **cuts** (AB) & (RB) **in** H & G **respectively. (complete the figure)**

   a) Use a pair of triangles to prove that $AH\times CR=4\ CA$. **(1pt)**

   b) Use **Thales' property** in another set of triangles to prove that: $CE^2=CG\times CH$. **(1pt)**

   c) Deduce that the product $CG\times CH$ is constant. **(¾pt)**

5th exercise: (6½pts)

Consider a **right isosceles** triangle **AOB** at **O**, so that **AB = 6 cm**, and **K** is the midpoint of [**AB**].

1) Draw a clear figure. **(½pt)**
2) Calculate the lengths of the segments [**OA**] and [**OK**]. **(1pt)**
3) Trace the circle (**C**) of center **O** and radius [**OK**] and prove that (**AB**) is tangent to (**C**). **(½pt)**
4) **The circle (C) intersects [OA] at P and [OB] at L.**

   a) What is the nature of the triangle **OPL**? **Justify. (¾pt)**

   b) Show that (**PL**) is parallel to (**AB**). **(¾pt)**

   c) Find the length of segment [**PL**] by **applying Thales' Theorem**. **(1pt)**
5) (**OB**) cuts (**C**) at **E** and (**OA**) cuts (**C**) at **F**.

   Determine the nature of the quadrilateral **PEFL**. **Justify. (1pt)**
6) **The tangent to (C) at E intersects (AB) at S.**

   What is the nature of the triangle **SEK**. Then calculate the measure of each angle in the triangle **SEK**. **(1pt)**

Good work