الإسم: الرقم:

مسابقة في الرياضيات الانكليزي المدّة: ساعتين

. ارشادات عامة: – يسمح باستخدام آلة حاسبة غير قابلة للبرمجة او اختزان المعلومات او رسم البيانات.

ليستطيع المرشح الاجابة بالترتيب الذي يناسبه دون الالتزام بترتيب المسائل الواردة في المسابقة.

عدد المسائل 6 وجميعها إلزامية.

العلامة القصوى .30

## 1st exercise: (4 ½ pts)

1) Given the following two numbers:

$$A = \left(\frac{3}{8}\right)^2 - \frac{1}{8}$$
;  $B = \left(3 - \sqrt{5}\right)^2 + 2\left(25 + \sqrt{45}\right)$ 

By writing all the steps of calculations:

a- Calculate A and show that  $A = \left(\frac{1}{8}\right)^2$ . (3/4 pt)

b- Calculate B and show that  $B = 8^2$ . (34 pt)

c- Deduce that  $A \times B = 1$ . (½ pt)

2) Let x b e a real number defined by:  $x = \sqrt{8 - 2\sqrt{7}} - \sqrt{8 + 2\sqrt{7}}$ .

a- Compare  $8-2\sqrt{7}$  and  $8+2\sqrt{7}$ . Deduce the sign of x. (1pt)

b- Calculate  $x^2$  and show that  $x^2 = 4$ . Deduce the value of x. (1 ½ pts)

# $2^{nd}$ exercise: $(2 \frac{1}{2}$ pts)

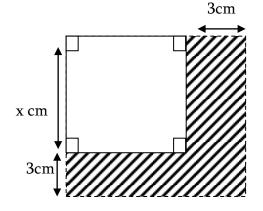
Consider a square whose side measures x cm. the side is increased by 3cm.

1) Determine, among the 3 algebraic expressions given below, the expression that expresses the increase in area of the square:

$$(x^2+3^2)^2-x^2$$
;  $x^2-(x+3)^2$ ;  $(x+3)^2-x^2$ . (1pt)

2) Show that the obtained expression can be written as: 3(2x+3).

Deduce the value of x when the increase in area is  $33\text{cm}^2.(1 \frac{1}{2} \text{ pts})$ 



### $3^{rd}$ exercise: $(3 \frac{1}{2} pts)$

Consider the two polynomials:  $f(x) = 4x^2 - 1 + (2x - 1)^2 - (2 - 4x)(x + 2)$ ;  $g(x) = (3x + 1)^2 - (x + 2)^2$ 

- 1) Develop and reduce f(x). ( $\frac{3}{4}$  pt)
- 2) Show that f(x) = 2(2x-1)(3x+2). (3/4 pt)
- 3) Write g(x) in the form of a product of two factors of the first degree. ( $\frac{3}{4}$  pt)
- 4) Let  $h(x) = \frac{f(x)}{g(x)}$ 
  - a- For what values of x, h(x) is defined. (  $\frac{1}{2}$  pt)
  - b- Simplify h(x), then solve the equation h(x) = 2. (3/4 pt)

### 4th exercise: (7pts)

A video-club proposes two options to its clients:

Option A: pay a sum of 40\$ each month and 2\$ for each videotape rented.

Option B: pay 12\$ for each videotape rented.

1) Let:

x be the number of videos rented,

 $Y_A$  be the sum paid in option A,

 $Y_{B}$  be the sum paid in option B.

- a- Express in each of the two options, the sum paid in terms of x. (1pt)
- b- Recopy and complete the following table: (1pt)

X	3	4	5
$Y_{_A}$			
$Y_{\scriptscriptstyle B}$			

2) Represent graphically these two options in the same system.

(1cm on the x-axis represents 1 video tape; 1 cm on the y-axis represents 10\$) (1 ½ pts)

- 3) What is the advantageous option if the client wants to rent in a month:
  - a. 3 videotapes? ( $\frac{1}{2}$  pt)
  - b. 5 videotapes? (½ pt)
- 4) Determine the value of x, so that option A is more advantageous than option B. (1pt)
- 5) A person chooses option B and paid 156\$. How many videotapes did he rent? Is he right for choosing option B? Justify. (1 ½ pts)

### $5^{th}$ exercise: $(5 \frac{1}{2} \text{ pts})$

ABC is an isosceles triangle at vertex A. The altitude issued from A cuts [BC] at H. (see figure below)

Given BC = 6cm and AH = 4cm.

Let M be a point of [BH] such that BM = x. The parallel to (AH)through M cuts (AB) at P and (AC) at Q.

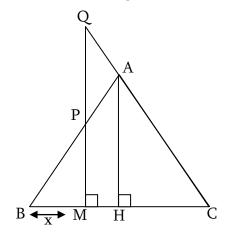
1) a- Calculate BH and give an encirclement for x. (1pt)

b- Show that:  $\frac{MP}{AH} = \frac{x}{3}$ . Deduce MP in terms of x. (1pt)

2) a- Express MC in terms of x. (½ pt)

b- Show that MQ =  $\frac{4}{3}(6-x)$ . (1pt)

- c- Find the value of x so that MQ = 3MP. (1pt)
- d- In this case, precise the position of P on [AB]. (1pt)



# 6<sup>th</sup> exercise: (7pts)

In an orthonormal system x'ox, y'oy, consider the two straight-lines (d) and (d') of respective equations (d): y = -2x + 5 and (d'):  $y = \frac{1}{2}x$  and the two points A(-1;7) and B(-2;-1)

- 1) Draw (d) and (d') then plot A and B. (1 ½ pts)
- 2) Show that the point I(2,1) is the point of intersection of (d) and (d'). ( $\frac{3}{4}$  pt)
- 3) Verify by calculation that A belongs to (d) and that (d') passes through B. (1pt)
- 4) Given: AI =  $\sqrt{45}$ ; BI =  $\sqrt{20}$ ; AB =  $\sqrt{65}$ .
  - a- Deduce that the triangle ABI is right at I. ( 3/4 pt)

b- Let J be the center of the circle circumscribed about the right triangle ABI. Calculate the radius of the circle and determine the coordinates of J. (1pt)

- 5) a- Write the equation of the straight-line ( $\Delta$ ) passing through J and parallel to (d'). (1pt)
  - b- ( $\Delta$ ) cuts (d) at L. Find the coordinates of L. (1pt)