

- يسمح بإستعمال آلة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على المسائل بالترتيب الذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 30
- عدد المسائل: 4

### 1<sup>st</sup> exercise: (7 ½ pts)

In this exercise, the five parts are independent. (1 ½ pts for each part)

- 1) Let  $E = (\sin x - \cos x - 1) \cdot (\sin x - \cos x + 1)$ . Calculate E.
- 2) The table below shows the ages of 40 persons.

|           |    |    |    |    |    |
|-----------|----|----|----|----|----|
| Age       | 15 | 16 | 17 | 18 | 19 |
| Frequency | 9  | 11 | x  | 9  | y  |

Calculate x and y knowing that the mean age is 16.8.

- 3) Consider the two polynomials:

$$P(x) = 9x^3 + 9x^2 + (c - 1)x + 5$$

$$Q(x) = (a^2 - b^2)x^3 + (a + b)x^2 + 2d - 1$$

Determine a, b, c and d so that P(x) and Q(x) are identical.

- 4) Consider in an orthonormal system (x'ox), (y'oy) the points E(3; 2), F(5; - 3); and G(1; 4). Determine the coordinates of the point H such that EFGH is a parallelogram.

(Note: you can draw a sketch if you want)

- 5) a) Given the real number  $T = \frac{\sqrt{45} + \sqrt{20} - 3\sqrt{5} + \sqrt{36}}{\sqrt{125} - \sqrt{80} + \sqrt{9}}$ .

Show that T is an integer.

- b) Deduce that T is a solution of the inequality  $5(x + 1) - 3\frac{x}{2} - 1 > \frac{x}{2} - 5$

### 2<sup>nd</sup> exercise: (5 ½ pts)

Upon studying the type of cellular phones used by each of the 25 students of grade 9, we obtained the following results organized in the table below:

|                        |       |        |            |         |
|------------------------|-------|--------|------------|---------|
| Type of cellular phone | Nokia | iphone | Blackberry | Samsung |
| Frequency              | x + 1 | 4x + 1 | x + 3      | 3x + 2  |

(x is a natural number)

- 1) Determine the character and its nature. ( ½ pt)
- 2) Show that x = 2, then determine the most used cellular phone. (1pt)
- 3) a) Set up the table of frequencies and the central angles in degrees. (1 ½ pts)  
b) Draw the semi-circular diagram for this statistical distribution. (1pt)
- 4) Can you determine the increasing cumulative frequencies? Justify. ( ¾ pt)
- 5) Calculate the percentage of the students who have Blackberry phones. ( ¾ pt)

### 3<sup>rd</sup> exercise: (11pts)

In the plane of an orthonormal system  $(x'ox)$ ,  $(y'oy)$ , where the unit of length is the centimeter, consider the straight-line  $(d)$  of equation  $y - 2x = -4$  and the point  $H\left(\frac{a}{5}; \frac{-a}{10}\right)$  ( $a$  is an integer).

- 1) Draw the straight-line  $(d)$ . (  $\frac{1}{2}$  pt)
- 2) a) Calculate  $a$  so that  $H$  belongs to  $(d)$ . Plot  $H$ . (  $\frac{3}{4}$  pt)  
b) Let  $H\left(\frac{8}{5}; \frac{-8}{10}\right)$ , show that  $H$  is the orthogonal projection of  $O$  and  $(d)$ . (1pt)
- 3) a)  $(d)$  cuts  $(x'ox)$  at  $A$  and  $(y'oy)$  at  $B$ . Calculate the coordinates of  $A$  and  $B$ . (  $\frac{1}{2}$  pt)  
b) Determine  $\widehat{COA}$  and  $\widehat{SOBH}$  in terms of  $OH$  only. (1  $\frac{1}{2}$  pts)  
c) Show that  $\widehat{AOH}$  and  $\widehat{OBH}$  are equal. (  $\frac{3}{4}$  pt)  
d) Let  $\widehat{AOH} = \widehat{OBH} = \alpha$  and use **part b** to show that  $OH = \frac{4\sqrt{5}}{5} \text{ cm}$ . (1pt)
- 4) a) Calculate to the nearest  $10^{-2}$  degree the angle  $\widehat{OAB}$  using the equation of  $(d)$ . (  $\frac{3}{4}$  pt)  
b)  $(d_2)$  is a decreasing straight-line passing through  $A$  and making an acute angle of  $60^\circ$  with  $(x'ox)$ . Determine an equation of  $(d_2)$ . (  $\frac{3}{4}$  pt)
- 5) Given the point  $C(0; 2)$ . Designate by  $I$  the midpoint of  $[AB]$ .  
a) Using the points of the figure, complete  $\vec{CA} + \vec{CB} = \dots$  (  $\frac{1}{2}$  pt)  
b) Let  $G\left(\frac{2}{3}; -\frac{2}{3}\right)$ , calculate the coordinates of  $I$ , then prove that the points  $C$ ,  $G$ , and  $I$  are collinear. (1  $\frac{1}{2}$  pts)  
c) Using the coordinates of the points  $C$ ,  $G$  and  $I$ , show that  $\vec{GI} = \frac{1}{3}\vec{CI}$ . What can you deduce about the point  $G$ ? (1 $\frac{1}{2}$ pts)

### 4<sup>th</sup> exercise: (6pts)

Consider a circle  $(\sigma)$  of center  $O$  and diameter  $[BC]$  such that  $BC = 10\text{cm}$ . Let  $A$  be a variable point of  $(\sigma)$  distinct of  $B$  and  $C$ . The perpendicular issued from  $A$  to  $(BC)$  cuts  $[BC]$  at  $H$  and the circle  $(\sigma)$  at  $D$  such that  $BD < AC$ .

- 1) Draw a figure. (  $\frac{1}{2}$  pt)
- 2) a) Show that the two triangles  $HBD$  and  $HAC$  are similar and write the ratio of similarity. (1pt)  
b) Verify that the triangle  $HAC$  is an enlargement of triangle  $HBD$  where the center of enlargement is to be determined. (  $\frac{3}{4}$  pt)
- 3) **(AB) and (CD) meet at E.**  
a) Is the quadrilateral  $ABDC$  cyclic? Justify. (  $\frac{1}{2}$  pt)  
b) Show that  $\widehat{EBD} = \widehat{ACD}$ . (  $\frac{3}{4}$  pt)  
c) Show that the 2 triangles  $EBD$  and  $EAC$  are similar then show that  $EB \times CA = EC \times BD$ . (1pt)
- 4) Let  $M$  be the point defined by  $\vec{CM} = \vec{AB} + \vec{AC}$ .  
a) Locate the point  $M$ . (1pt)  
b) Show that  $\vec{CM} = 2\vec{AO}$  then determine the locus of the point  $M$  as  $A$  describes the circle  $(\sigma)$ . (1 $\frac{1}{2}$  pts)