| مسابقة في الرياضيات الانكليزي | الرإسم : الردّة : ساعتين |
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! إرشادات عامة:
يسمح بإستعمال آلة حاسبة غير قابلة للبرمجة
يكن الإجابة على المسائل بالترتيب الذي تريد

$$
\begin{aligned}
& \text { يرجى الإجابة بخط واضح ومرتب } \\
& \text { العلامة القصوى من } 30 \\
& \text { عدد المسائل: } 6
\end{aligned}
$$

## $1^{\text {st }}$ exercise: ( 7 pts )

In the following table, only one of the proposed answers to each question is correct. Write the number of each question and the corresponding answer, and justify.

| No. | Questions | Answers |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | A | B | C |
| 1. | The measure of the acute angle that straight-line (d) of equation $y=\sqrt{3} x+2$ makes with $y^{\prime} o y$ is equal to | $60^{\circ}$ | $30^{\circ}$ | $45^{\circ}$ |
| 2. | The integers that are solutions of the following system $\left\{\begin{array}{l}\frac{3 x+1}{3}-\frac{x-1}{2}<2 x-1 \\ 3(x-1)+2 \leq 2 x+3\end{array}\right.$ | 1,2, 3 | 2, 3 | 2, 3, 4 |
| 3. | In the two sections A and B of grade 9, you are given: <br> Section A: 20 students, average of marks is 15 Section B: 30 students average of marks is 10 The average of marks of the 50 students is equal to: | 12.5 | 12 | 11 |
| 4. | Given $\quad A=\frac{2 \times\left(\operatorname{Cos} 60^{\circ}+\operatorname{Sin} 45^{\circ}\right)}{\tan 60^{\circ}} \times(\sqrt{6}-\sqrt{3})$ without using the calculator and showing all the details of calculations then $\mathrm{A}=$ | $\tan 45^{\circ}$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ |
| 5. | Given in an orthonormal system the straight-line (d) of equation $y=\frac{4}{3} x+4$. The equation of ( $\mathrm{d}^{\prime}$ ), the image of (d) by the translation of vector $\vec{v}(3,-4)$ is | $y=\frac{4}{3} x-4$ | $y=\frac{4}{3} x$ | $y=4 x$ |

## $2^{\text {nd }}$ exercise: ( $31 / 2 \mathrm{pts}$ )

In the adjacent figure, consider a semi-circle (C) of diameter [DC] and a rectangle $A B C D$.
M is a point of (C) such that $D M=\pi, \mathrm{MC}=1, \mathrm{AB}=\mathrm{x}$ and $\mathrm{BC}=\mathrm{x}-2$. $(\mathrm{x} \succ 2)$ (The unit of length is the cm ).

1) Determine the area $A$ of the rectangle $A B C D$ in terms of $x .(1 / 2 p t)$
2) Show that the area $\mathrm{A}^{\prime}$ of the shaded region can be expressed by $\mathrm{A}^{\prime}=\frac{\pi}{2}\left(\frac{x^{2}}{4}-1\right) \cdot\left(1^{11 / 2} \mathrm{pts}\right)$

3) a) Verify the following equality: $3 x^{2}-8 x+4=(x-2)(3 x-2) \cdot(1 / 2 \mathrm{pt})$
b) Can you calculate the value of x so that the area of the shaded region A ' is equal to $\frac{\pi}{2} \times \mathrm{A}$ ? Justify. (1 pt)

## $3^{\text {rd }}$ exercise: (4 pts)

In a factory A, the monthly salary of a secretary is $1000 \$$, that of a technician is $1500 \$$ and that of a director is 2000\$.
Factory A:

| Salary in \$ | 1000 | 1500 | 2000 | Total |
| :--- | :---: | :---: | :---: | :---: |
| Frequency | 3 |  |  | 10 |
| Central angles |  | $180^{\circ}$ |  |  |

1) Copy the table and complete it by showing all the details of calculation. ( $11 / 2 \mathrm{pts}$ )
2) Determine the population, the character, the values and the nature of the character. ( 1 pt )
3) In another factory B, the secretary gets a salary reduced by $20 \%$ than that of a secretary in the factory A while the technicians get the same salary.
Consider the following table of frequency in factory B.

| Salary in \$ | $\mathbf{x}$ | 1500 | $\mathbf{y}$ |
| :--- | :---: | :---: | :---: |
| Frequency | 8 | 9 | 3 |

Calculate $\mathbf{x}$ and $\mathbf{y}$ knowing that the mean salary in factory $\mathbf{B}$ is $1340 \$$ and that the number of directors is 3 . ( $11 / 2 \mathrm{pts}$ )

## $4^{\text {th }}$ exercise: ( $61 / 2 \mathrm{pts}$ )

In an orthonormal system (x'ox, y'oy), consider the points $A(-2 ; 5), B(5 ; 3), C(\mathbf{a} ; 2 \mathbf{a}+3), E(1 ; 1)$ and the straight-line $\left(D_{1}\right)$ of equation $2 \mathrm{y}=3 \mathrm{x}$.

1) Plot the points $\mathrm{A}, \mathrm{B}, \mathrm{E}$ and draw $\left(D_{1}\right) \cdot\left(1 \frac{1}{2} \mathrm{pts}\right)$
2) Calculate a so that the point $C$ belongs to the median issued from $A$ in the triangle $A B E$. ( $11 / 2 \mathrm{pts}$ )
3) Calculate $m$ and $n$ so that the straight-line (d) of equation ( $m-2$ ) $x+(2 n-3) y+15=0$ passes through the point A and be perpendicular to $(\delta): \mathrm{y}=2 \mathrm{x}-5$. ( $1^{11 / 2} \mathrm{pts}$ )
4) Let H be the point defined by $\overrightarrow{E A}+\overrightarrow{E B}=\overrightarrow{E H}$.
a) Determine the nature of the quadrilateral EBHA. ( $1 / 2 \mathrm{pt}$ )
b) Calculate the coordinates of H. (1 pt)
c) Complete: H is the translate of B by the translation of vector $\ldots$ because $\ldots \ldots . . . . . . .(1 / 2 \mathrm{pt})$

## $5^{\text {th }}$ exercise: ( 3 pts )

Tarek wants to install at home a basket of basketball. He should fix it to 3.05 m of the ground .The ladder that he uses measures 3.20 m .

1) Calculate the acute angle formed by the ladder and the ground. (Give the answer rounded to the nearest degree.(1pt)
2) If the ladder slides on the wall until it is flat on the ground, then determine the locus of the midpoint $I$ of the ladder $(1 \mathrm{pt})$.
3) Without calculating BC , use complementary angles to
 calculate $\cos \hat{B A C}$ then deduce an approximate value for $B \hat{A} C$ to the nearest 0.01 by excess .(1 pt)

## $6^{\text {6th }}$ exercise: ( 6 pts )

$(\mathrm{C})$ is a circle of center O and diameter $[\mathrm{AB}]$.
The straight-line (d) is tangent to (C) at B and $M$ is a variable point of (d). The perpendicular to (AM) passing through A cuts (d) at N.

1) Draw a figure. $(1 / 2 \mathrm{pt})$
2) a) Show that $\hat{B A} N=A \hat{M} B \cdot(3 / 4 \mathrm{pt})$
b) Using $\tan \hat{B} A N$ and $\tan A \hat{M B}$ in two convenient right triangles, show that $B M \times B N=4 R^{2} .(1 \mathrm{pt})$
3) The parallel to (AN) through $O$ cuts ( BN ) at F and (AM) at E. Show that F is the midpoint of $[\mathrm{BN}] .(3 / 4 \mathrm{pt})$
4) (AM) cuts the circle (C) at I.
a) Show that the quadrilateral AIBN is a right trapezoid. $(3 / 4 \mathrm{pt})$
b) Show that the two triangles AIB and FOB are similar then write the ratio of similarity. ( $11 / 4 \mathrm{pts}$ )
5) Show that the quadrilateral AEBF is inscribed in a circle whose center $S$ and radius are to be precised.(1pt)
