ليسه دي زار **الشهادة المتوسطة** التجربة الثالثة لعام 2013 – 2014

الرقم :	الإسم :	المدّة : ساعتان	مسابقة في الرياضيات الإنكليزي

إرشادات عامة:

- يسمح بإستعمال آلة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على المسائل بالترتيب االذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 30

1st exercise: (6½ pts)

In the following table, only one of the proposed answers to each question is correct. Write the number of each question and the corresponding answer, and **justify**.

<table>
<tr><th rowspan="2">No.</th><th rowspan="2">Questions</th><th colspan="3">Answers</th></tr>
<tr><th>A</th><th>B</th><th>C</th></tr>
<tr><td>1.</td><td>If x, y and z are respectively proportional to 5, 7, 9 and $2x - y + 3z = 180$ then **(1½ pts)**</td><td>$x + y + z = 126$</td><td>$x + y + z = 130$</td><td>$x + y + z = 90$</td></tr>
<tr><td>2.</td><td>In a rectangle, given **L**: length, **W**: width, $\mathcal{A}$: area. If a new rectangle is obtained by **enlarging L** by 20% and **reducing W** by 20% then a ***new area*** $\mathcal{A}'$ is obtained. So, the new rectangle is : **(1½pts)**</td><td>A reduction of the old rectangle by 4%</td><td>An enlargement of the old rectangle by 4%</td><td>Congruent to the old rectangle</td></tr>
<tr><td>3.</td><td>If x is a real number in the interval $[-2\,;5[$ **and** also $-8 \le x < 5$, then x belongs to **(1 pt)**</td><td>$[-2\,;5[$</td><td>$]-2\,;5[$</td><td>$]-2\,;5]$</td></tr>
<tr><td>4.</td><td>The couple solution of the system $\begin{cases} 6\cos(\alpha) - 4\sqrt{2}\sin(\beta) = -1 \\ 2\cos(\alpha) + \sqrt{2}\sin(\beta) = 2 \end{cases}$ **is:** **(1½ pts)**
(Note: Solve the system where α and β **are acute angles)**</td><td>$\alpha = 60^\circ$
and
$\beta = 45^\circ$</td><td>$\alpha = 45^\circ$
and
$\beta = 60^\circ$</td><td>$\alpha = 30^\circ$
and
$\beta = 45^\circ$</td></tr>
<tr><td>5.</td><td>Consider the following statistical distribution:<table><tr><td>Values x_i</td><td>x_1</td><td>x_2</td><td>x_3</td><td>x_4</td></tr><tr><td>Frequencies n_i</td><td>n_1</td><td>n_2</td><td>n_3</td><td>n_4</td></tr></table>If y is a new variable such that $y_i = 10\,x_i - 4$ and $\bar{x}$ is the mean of the old variable x then the mean $\bar{y}$ of the new variable will be
(Note: Show your detailed justification) **(1 pt)**</td><td>$\bar{y} = 10\,\bar{x} - 4$</td><td>$\bar{y} = 10\,\bar{x}$</td><td>$\bar{y} = \bar{x}$</td></tr>
</table>

2nd exercise: (5½ pts)

1) α is the measure of an acute angle such that cos(α) = **m**.

Justify which of the following propositions below is correct about **m**:

a) $m = \dfrac{4\sqrt{2}}{9}$ b) $m = -\dfrac{4\sqrt{2}}{9}$ c) $m = \dfrac{9}{4\sqrt{2}}$ **(1 pt)**

2) Deduce the numerical values of sin(α) and tan(α).**(1½ pts)**

3) If (d) is a decreasing straight-line in an orthonormal system $(x'Ox\,;y'Oy)$ passing through the point $D(-1;4)$ and making an acute angle α with the $x-axis$, then determine the equation of (d). **(1pt)**

4) Without using the calculator, show that: $A = \dfrac{2\,(\cos 60^\circ + \sin 45^\circ)}{\tan 68^\circ \times \tan 22^\circ} \times \left(\sqrt{6} - \sqrt{3}\right)$ and $B = \dfrac{\sqrt{3}}{3}$ are reciprocals.**(2pts)**

3rd exercise: (3½ pts)

Upon studying the **expenses** of a family that spends **1 750 000 L.L** monthly, the following results were obtained:

30% for food, **10%** for transportation, **20%** for housing, **8%** for clothing, **11%** for energy, **15%** for schooling, and **6%** for entertainment.

1) Determine the population, the character, the modalities and the nature of the character. **(1pt)**
2) Make a table of frequencies and represent this data in a bar diagram of frequencies in (%). **(1½ pts)**
3) Can you determine the **increasing cumulative frequencies** of this statistical distribution? **Justify. (1pt)**

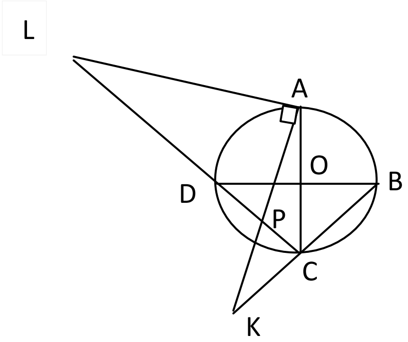
4th exercise: (9 pts)

In an orthonormal system of axes $(x'Ox ; y'Oy)$, consider the points: $A(1;3)$, $B(4;-1)$ and $D(5;6)$.

1) a) Place the given points A, B and D in the orthonormal system. **(1pt)**
   b) Determine **graphically** the coordinates of $\overrightarrow{AD}$ **(leave the traces on your figure)** , then calculate the coordinates of the point C such that ABCD is a parallelogram. **(1¼ pts)**
   c) Given that $C(8;2)$, then show, by calculation, that **(BD) and (AC) are perpendicular**. **(¾ pt)**
   d) Show that ABCD is a **square. (1pt)**
2) **Consider the two circles** $\Omega(A;2)$ and δ (B; 3) that intersect at the point E.
   a) Draw the two circles (Ω) and (δ) and the point E. **(½pt)**
   b) Show that the two circles (Ω) and (δ) are tangent externally at E. **(1pt)**
3) Given the straight-line (Δ) of equation $4y + 5x + 7 = 0$.
   a) Which axis does the straight-line (Δ) cut: the **positive or the negative** y- axis? **Justify. (½ pt)**
   b) Determine the equation of the straight-line (Ψ) that is parallel to (Δ) and passes through B. **(¾ pt)**
   c) Give **only** a plan made of 3 steps to how you can calculate the distance between the two parallel straight -lines (Δ) and (Ψ). **(¾ pt)**
4) Suppose that **G is a variable point** that describes the circle $\Omega(A;2)$ and **H is the image of G** by the translation of vector $\overrightarrow{AB} + \overrightarrow{AD}$. Show that the point H varies on a circle whose center and radius are to be determined. **(1½ pts)**

5th exercise: (5½ pts)

Consider in the figure to the right:



- **(C) is a circle of center O and radius** $r = \frac{a\sqrt{2}}{2}$ **where** $a > 0$.
- **[AC] and [BD] are two perpendicular diameters of (C).**
- **K is a point on (BC) such that** $CK = b,\ b > 0$.
- **The perpendicular passing through A to (AK) cuts (CD) in L.**
- **[LC] and [AK] intersect at P.**

1) a) What is the nature of the quadrilateral ABCD? **(¾pt)**
   b) Prove that the triangle ADO is right isosceles then use trigonometry to prove that AD = a.**(1½pts)**
2) Show that the points A, L, C and K belong to the same circle (Ω) whose center S is to be determined. **(¾pt)**
3) Use the property of Thales in the two triangles ADP and CPK to prove that $PD = \frac{a^2}{a+b}$ **(1 pt)**
4) a) Show that the two triangles ALD and ADP are similar then write the ratio of similitude. **(1pt)**
   b) Deduce that LD = $a + b$. **(½ pt)**