مسابقة في الرياضيات الإنكليزي المدّة : ساعتان الإسم : الرقم :

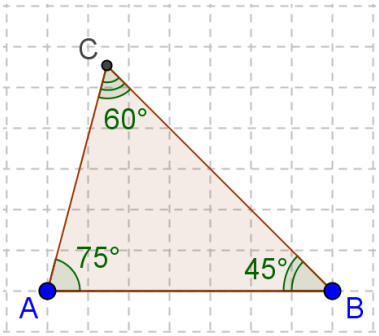
**إرشادات عامة:**

- يسمح بإستعمال ألة حاسبة غير قابلة للبرمجة **للتأكد من الاجابة فقط**
- يمكن الإجابة على ألمسائل بالترتيب االذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 40

**1st exercise: (7 ½ pts)**

The adjacent figure represents the triangle ABC such that:

$\hat{BAC} = 75^0$, $\hat{ABC} = 45^0$ and $\hat{ACB} = 60^0$.




**We suppose that:** $a = \cos\hat{A}$, $b = \cos\hat{B}$, $c = \cos\hat{C}$ and $\cos(75^0) = x$ .

**We admit that *a*, *b* and *c* verify the relation :** $a^2 + b^2 + c^2 + 2abc = 1$.

1) Use **the above given relation** and **the table of particular angles,**
show that: $4x^2 + 2x\sqrt{2} - 1 = 0$. **(1pt)**

2) Consider the real numbers: $U = \dfrac{\sqrt{6}-\sqrt{2}}{4}$ and $V = \dfrac{-\sqrt{6}-\sqrt{2}}{4}$.

a) Verify that U and V **are two solutions of the equation:** $4x^2 + 2x\sqrt{2} - 1 = 0$ **.(1½pts)**

b) Deduce the **exact value of** $\cos(75^0) = x$ and then deduce sin(75⁰) and sin(15⁰). **(¾pt,1pt, ½ pt)**

3) a) Compare the numbers: $\dfrac{\sqrt{6}+\sqrt{2}}{4}$ and $\dfrac{\sqrt{2+\sqrt{3}}}{2}$ **.(1pt)**

b) Deduce that $\tan(75^0) = 2 + \sqrt{3}$, and then calculate $\cot(75^0)$ **.(1pt,¾pt )**

**2nd exercise: (12 pts)**

In an orthonormal system of axes $(x'Ox, y'Oy)$, **where the unit is cm,** consider the points A(0;2), B(-2,;0), C(4;-2) and D(4*m*- 2; 4*m*), **where *m* is a real number.**

(d) is the straight line of equation: $y = -3x - 8$.

1) a) Place the points A, B, C and draw (d) in the orthonormal system. **(¾pt, ½pt)**

b) Show that the straight line (AB) is defined by the equation: $y = x + 2$. **(¾pt)**

2) **Let I be the midpoint of [BC].**
Calculate AI and BC **then deduce** that ABC is right at A.**(1¾pts)**

3) a) Show that **as *m* varies**, the point D describes **the fixed straight line** (AB).**(¾pt)**

b) **Calculate *m*** so that [CA) is the angle bisector of $\widehat{BCD}$ .**(1pt)**

4) Calculate the area of (C), **the circle circumscribed about the right triangle ABC. (¾pt)**

5) **(d) is tangent to (C) at a point E(*a* ; *b*).**

a) Calculate, **in terms of *a* and *b*,** the slope of the straight line (EI).**(¾pt)**

b) Show that the coordinates of the point E, verify the following system: $\begin{cases} a - 3b = 4 \\ 3a + b = -8 \end{cases}$ **.(1½pts)**

c) Solve the system then deduce the coordinates of E. **(1pt)**

6) **H is the point such that:** $\overrightarrow{CB} + \overrightarrow{CD} = \overrightarrow{CH}$ **. In this question take *m* = 1.**

a) Calculate the coordinates of H and determine the nature of the quadrilateral BCDH.**(1½pts)**

b) Determine the coordinates of J, the center of the circle (C'), the image of the circle(C) by the translation of vector $\overrightarrow{CA}$ .**(1pt)**

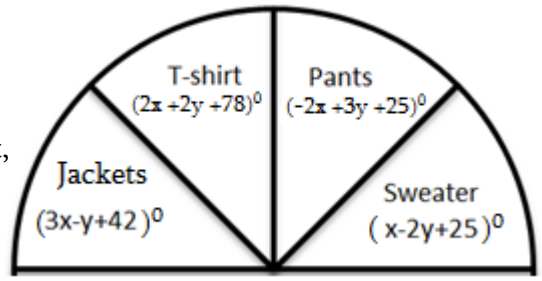
**3rd exercise: (6 ½ pts)**

In a **clothes store**, a study was conducted about the **types of 20 pieces of clothes.** The following table shows the obtained data.

| Clothes | Pants | Sweater | T-shirt | Jackets | Total |
|---|---|---|---|---|---|
| Frequencies | $2x - y + 4$ | $-2xy + 3y + 10$ | $x + y - 1$ | $2xy - 2$ | 20 |

**Part A** :

The adjacent **semi-circular diagram** gives the distribution of the above modalities.




1) **For the above study,** precise the population, the statistical unit, the character and its nature. **(1pt)**

2) a) **Explain how** the following equations: $x + y = 3$ and $2x + y = 5$ were obtained. **(1½pts)**

b) Calculate the numerical values of $x$ and $y$. **(¾pt)**

3) Construct the bar diagram of frequencies. **(¾pt)**

**Part B**:

Now, we want to **study the shoe size** of the shoes bought by **80 customers.**

For this, we consider the following incomplete table.

| Shoe size | 37 | 38 | 41 | 42 | Total |
|---|---|---|---|---|---|
| Frequency | 5 | | 17 | | |
| **Increasing Cumulative Frequencies in %** | | 33.75% | | | |

1) a) Interpret the ICF % of the shoe size 38. **(¼pt)**

b) Reproduce the above table and complete the missing numbers. **(1½pts)**

2) Calculate **the mean** shoe size. **(¾pt)**

**4th exercise: (6pts)**

(S) is a semi-circle of center O and radius "$r$" where [AB] is a diameter. Let C be the point on (S) such that $\overset{\frown}{BC} = 2\overset{\frown}{AC}$. The perpendicular to (OC) at O cuts (S) at D.

1) a) Show that $\overset{\frown}{AC} = 60^0$, then draw the semi-circle(S) and place the point C.**(¾pt, ½pt)**

b) Calculate the measure of the arc $\overset{\frown}{BD}$ and show that triangle ABC is semi equilateral. **(¾pt,¾pt)**

c) Use **trigonometric ratios (Sine, cosine or Tangent)** to calculate, **in terms of $r$,** AD and BD. **(1½pts)**

2) The straight-lines (BC) and (AD) intersect at I.

a) Show that the triangles ACI and BDI are similar.**(1pt)**

b) Deduce that: $AC \times BI = AI \times BD$. **(¾pt)**

**5th exercise: (4¼ pts)**

During the holidays in the Greek islands, a man wants to rent a scooter. The owner offered him the following two options:

* **Option I**: Pay **3 € for every traveled kilometer** (km).
* **Option II**: Pay **a fixed price of 10 €** and then obtain **a reduction of 30% on the charge** for every traveled Kilometer in option I.

**Designate by:**

- $x$ the number of traveled kilometers.
- $f(x)$ **and** $g(x)$ the sum of money paid for option I and option II, **respectively.**

1) a) Express $f(x)$ in terms of $x$ and show that $g(x) = 2.1x + 10$.**(¾pt, 1pt)**

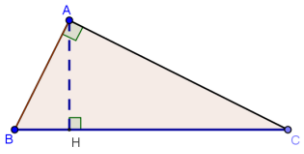
b) Which **option is more advantageous** for a man who is aiming to run the scooter for 13 km? **Justify.(1pt)**

2) Study **algebraically**, according to the values of x, the most advantageous option.

**(Note: two cases should be studied) (1½pts)**

**6th exercise: (3¾ pts)**

In the following table, **only one of the proposed answers** to each question is correct. Write the number of each question and the corresponding answer, with **justification**.

| No. | Questions | Answers | | |
|---|---|---|---|---|
| | | A | B | C |
| 1. | The system $\begin{cases} x^2-4 \geq x(x+4) \\ x^2 > -1 \end{cases}$, is true for all values of $x$ that belongs to the interval….. **(1½pts)** | $]-\infty;+\infty[$ | $]-\infty;-1[$ | $]-\infty;-1]$ |
| 2. | The measure of the **acute angle** that the straight line (d) of equation $\sqrt{3}y=-x+\sqrt{3}$ makes with **x'Ox** is equal to …… **(1¼pts)** | $60^o$ | $30^o$ | $120^o$ |
| 3. | In the right triangle ABC of hypotenuse [BC] and height [AH], we have BH = 3 and BC =15cm then $BA$ = ..... **(1pt)**   | $5\sqrt{3}$ | $3\sqrt{5}$ | $\sqrt{15}$ |