

الرقم :

الإسم :

المدة : ساعتان

مسابقة في الرياضيات الإنكليزي

إرشادات عامة:

- يسمح بإستعمال آلة حاسبة غير قابلة للبرمجة للتأكد من الإجابة فقط
- يمكن الإجابة على المسائل بالترتيب الذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 40

**1<sup>st</sup> exercise: (10pts)**Answer by **true** or **false** and **justify** your answer.

1) Given the system of equations (S):  $\begin{cases} 2a - 3b = -14\sqrt{2} \\ 3a + 2b = 18\sqrt{2} \end{cases}$  and the table (T) to the right:

|                 |                 |
|-----------------|-----------------|
| $\frac{a}{3}$   | $3 - 2\sqrt{2}$ |
| $3 + 2\sqrt{2}$ | $\frac{b}{8}$   |

If "a" and "b" verify the system (S), then (T) is a table of proportionality. (2pts)

2) If a boy downloaded 10% of a file from the internet **the first day** and then downloaded 20% **of the rest the second day**, so he will still has 70% of the **total** to download. (1pt)

3) If ABC is a triangle such that:  $AB = \frac{4^{1005} - 4^{1003}}{5 \times 4^{1003}}$ ,  $AC = \left( \frac{\sqrt{21} \times 10^{-12} \times \sqrt{63}}{21\sqrt{3} \times 10^{-13}} \right) - \sqrt{16}$  and

$BC = (\sqrt{3} - 1)^2 - \sqrt{16} + \sqrt{75}$ , then ABC is a semi-equilateral triangle at B. (2½pts)

4) The following identity is verified:  $\frac{\sin(\alpha)}{1 + \cos(\alpha)} + \frac{1 + \cos(\alpha)}{\sin(\alpha)} = \frac{2}{\sin(\alpha)}$  for any **acute** angle "α". (1½pts)

5) Is it true that **both solutions** of the equation:  $\frac{x}{\sqrt{5}-1} = \frac{\sqrt{5}-1}{x}$  ( $x \neq 0$ ) are solutions of:  $\frac{3x-5}{2} - \frac{x+3}{3} \leq 2$ ? (2pts)

6) If  $\vec{u}(2,3)$  &  $\vec{v}(a-2, a+1)$  are collinear then a=2. (1pt)

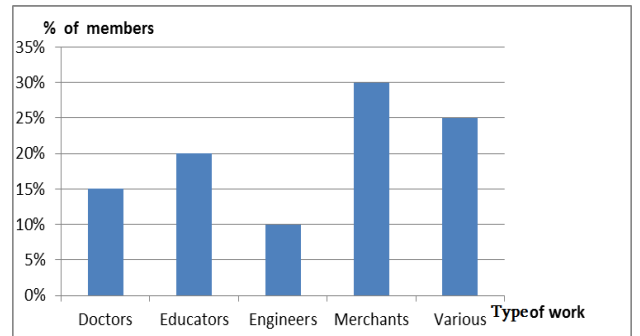
**2<sup>nd</sup> exercise: (9 pts)**

A public organization conducted a survey about the type of work that the members of certain families do.

**Part A:**

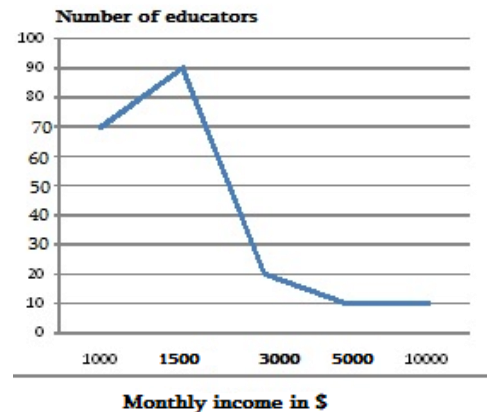
The following graph is obtained. (See it also on page 4)

- 1) Indicate the population, the variable and its nature. (¾pt)
- 2) a) What do the terms: doctors & merchants, on the given graph represent? (¾pt)
- b) What does each number on the **vertical axis** of the given graph represent? (¾pt)
- 3) **If the number of educators is 200**, then determine the total frequency. (¾pt)
- 4) Can you calculate the average value of the above data? **Justify**. (¾pt)

**Part B:**

Now to study the monthly income of the 200 educators, the organization constructed the following polygon.

- 1) Draw the increasing cumulative frequency table in percent that represents the adjacent graph. (1½pts)
- 2) **Is it true that:**
  - a) 50% of the educators have a monthly income of at least 3000\$? **Justify**. (1pt)
  - b) If all educators are to have the **same monthly income** it would be 2500\$? **Justify**. (1¼pts)
- 3) In July the monthly income of each educator **doubles and increases by 200\$**. Find the new average. (1½pts)



**3<sup>rd</sup> exercise: (12 pts)**

In an orthonormal system of axes  $x'Ox$  and  $y'Oy$ , consider the points A (- 4; 2), B (- 1; - 3), C (1; 5) and R (-3m; m - 4), and the straight line (d):  $5x + 3y + 14 = 0$ .

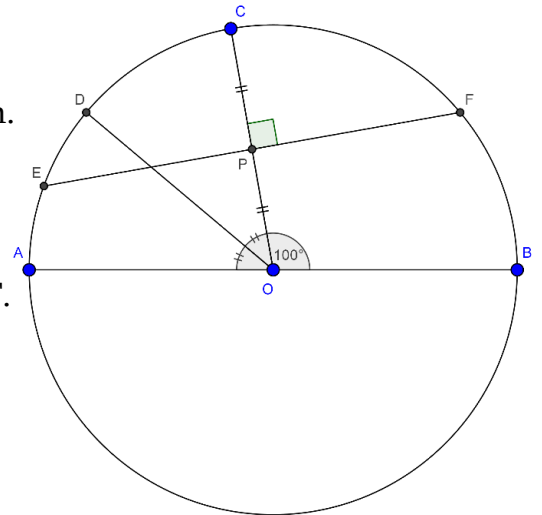
(m is real number such that  $m \neq -\frac{1}{3}$ )

- 1) a) Place the points A, B, and C in the system. (1pt)
  - b) Show that (d) passes through the points A and B. (¾pt)
  - c) Draw the (d) in the system. (½pt)
- 2) Write an equation of the straight line (AC). (¾pt)
- 3) a) Verify that the slope of the straight line (CR) is expressed by:  $a_{(CR)} = \frac{9-m}{3m+1}$ . (½pt)
  - b) Prove that if (CR) is perpendicular to (d), then A is the midpoint of [CR]. (1 ¼ pts)
- 4) **In what follows**, we give:  $m = 3$ .
  - a) What does the straight line (d) represent to the segment [CR]? Justify. (¾ pt)
  - b) Find the lengths of [AB] and [CR]. (1pt)
  - c) **Deduce** the nature of triangle CBA and the exact measure of [BC]. (1¼pts)
- 5) Let E be a point defined by:  $\vec{RE} = \vec{RA} + \vec{RB}$ 
  - a) What is the nature of quadrilateral ARBE? Justify. Show that E belongs to abscissa axis. (1¼pts)
  - b) Determine the nature of the quadrilateral ABEC? Justify. (1pt)
  - c) Deduce that the circle of center E and radius [AR] passes through the points B & C. (1pt)
- 6) Find an equation of ( $\Delta$ ), the image of (d) by translation of vector  $\vec{RB} + \frac{1}{2}\vec{RC}$ . (1pt)

**4<sup>th</sup> exercise: (9pts)**

The adjacent figure shows:

- ✓ (C) is a circle of center O, diameter [AB] and radius  $r = 4$  cm.
- ✓ D and C are two points such that:
  - $\widehat{BOC} = 100^\circ$  and (OD) is the bisector of the angle  $\widehat{COA}$ .
- ✓ P is the midpoint of [OC]
- ✓ The perpendicular bisector of [OC] that cuts (C) at E and F.



- 1) Reproduce the figure. (¾pt)
- 2) a) Prove that OCF is an equilateral triangle. (¾pt)
  - b) Calculate PF, **in two different ways**. (1¼pts)
  - c) Prove that the area of triangle AOC is  $8 \sin 80^\circ$ . (1pt)
- 3) **The parallel issued from O to (CD) cuts (BC) at I.**
  - a) Calculate the measures of the angles  $\widehat{BAC}$  and  $\widehat{DOA}$ . (1pt)
  - b) Deduce that CDOI is a parallelogram. (¾pt)
  - c) What is the image of D by the translation of vector  $\vec{OI}$ . (½pt)
  - d) What is the nature of the quadrilateral EDFI? Justify. (1pt)
- 4) **Let H is the orthogonal projection of C on (AB).**
  - a) Show that the triangles ABC and ACH are similar. (1pt)
  - b) Deduce AH and OH. (1pt)