مسابقة في الرياضيات الانكليزي	المدّة : ساعتان ونصف	الإسم :	الرقم :

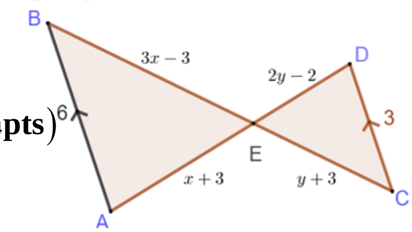
إرشادات عامة:

- يسمح بإستعمال آلة حاسبة غير قابلة للبرمجة.
- يمكن الإجابة على المسائل بالترتيب االذي تريد.
- يرجى الإجابة بخط واضح ومرتب.
- العلامة القصوى من 30

Exercise 1: (6½pts)

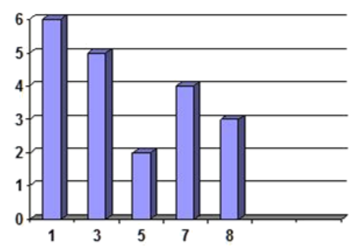
The following questions are independent:

1) If ABC is any triangle such that I is the midpoint of [BC], J is the midpoint of [AC] and G is its center of gravity, then show that $\overrightarrow{AB}+\overrightarrow{AC}=6\overrightarrow{GI}$. **(¾pt)**



2) knowing that (AB) and (CD) are parallel, calculate x and y. (y and x > 1) **(1¼pts)**

3) Find all negative integers that are solutions of the inequality: $\frac{3x+2}{5}-\frac{2x+1}{3}\leq\frac{x+4}{3}$. **(1pt)**



4) Determine the mean of the statistical series represented by the adjacent bar diagram. **(1pt)**

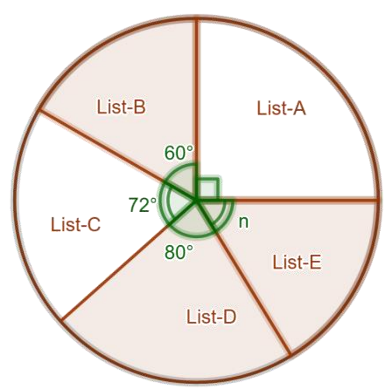
5) Given the following numbers $m=2^{12}-2^{10}$ and $n=\frac{1}{2^8-2^6}$

then calculate $m\times n$. **(1pt)**

6) Given the equation $x^2-ax+1=0$. Calculate a so that $x=\frac{7+\sqrt{125}+\sqrt{20}}{14}$ is a solution of the equation. **(1½pts)**

Exercise 2: (3 pts)

5-lists competed in the 2018 parliamentary elections in Beirut. A survey is conducted about the number of votes given to each list by 115 000 voters, and the following pie-chart was obtained.



1) Indicate the population, the variable and its nature. **(¾pt)**

2) **A list wins, if it gets more than 20% of the votes.**

 a) Does list-A win? Justify. **Determine how many votes this list took. (1pt)**

 b) What does "n" represent? Determine its value and interpret its meaning. **(¾pt)**

3) Can you calculate the average value of the above data? **Justify. (½pt)**

<u>Exercise 3: :</u> (11pts)

In an orthonormal system of axis $x'Ox$ and $y'Oy$, consider the points A (2; 0), B (6; 2) and C (-2; 2) and the straight line **(d):** $\mathbf{x - 2y - 2 = 0}$.

1) a) Plot the points **A, B, and C** in the system. **(1pt)**
   b) Show that the line **(d)** passes through the points **A** and **B** and then darw **(d). (¾pt)**
   c) Calculate the lengths **AB** and **AC**. Deduce the nature of triangle **ABC**. **(1pt)**
2) Let **(Δ)** be the line perpendicular to **(AC)** and passing through point **A .**
   Write an equation of **(Δ)**. **(¾pt)**
3) a) **Let (Δ') be the line parallel to** $y'Oy$ **and passing through B.**
   (Δ) and (Δ') intersect at point **E. Calculate the coordinates of E. (¾pt)**
   b) Show that the points A and B belong to the circle **(C)** of diameter **[CE]. (1pt)**
4) a) Prove that the coordinates of point **F the center of the circle (C)** are F**(2 ; 5). (½pt)**
   b) The circle **(C) cuts** $y'Oy$ in two points **L and K.** Calculate the coordinates of **L and K. (1½pts)**

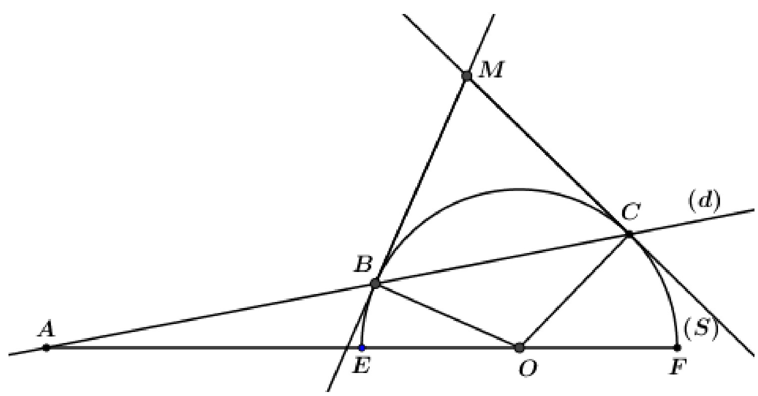
5) **Let S be the image of B by the translation of vector $\overrightarrow{AC}$.**
   a) Determine **<u>graphically</u>** the coordinates of vector $\overrightarrow{AC}$. (**Leave the traces on the figure**) **(½pt)**
   b) Determine the nature of quadrilateral ABSC, and then calculate the coordinates of point S. **(1¼pts)**
   c) Find the equation of the line (k) the image of (AB) by the translation of vector $\overrightarrow{AC}$. **(1pt)**

6) a) Calculate to the **nearest degree** the acute angle that the line **(AB)** makes with the axis **x'Ox**. **(½pt)**
   b) **Deduce** the acute angle that the line **(AB)** makes with the axis **y'Oy. (½pt)**

<u>Exercise 4:</u> (9½ pts)

In the adjacent figure:

- (S) is a semicircle of center O and radius R
- [EF] is the diameter of (S)
- A is the point on (EF) so that OA = 2 R
- (d) is a variable line through A that intersects (S) at B and C.
- The tangents at B and C to (S) intersect at M



1) Show that (OM) is the perpendicular bisector of [BC]. **(1pt)**
2) (OM) intersects [BC] at I.
   Let P be the orthogonal projection of M on (OA).
   a) Show that the triangles OIA and OMP are similar. **(1½pts)**
   b) Deduce the relation OA × OP = OM × OI. **(1pt)**
   c) By using the two **<u>right</u>** triangles **MPO** and **MOC** show that $\frac{OC}{\sin\widehat{CMO}} = \frac{OP}{\cos\widehat{MOP}}$. **(1½pts)**
3) a) Let (S') be the circle circumscribed about the triangle CIM. Show that (OC) is tangent to (S'). **(1pt)**
   b) Use two similar triangles to prove that OM ×OI = R^2. **(1½pts)**
   c) Calculate OP in terms of R. **(1pt)**
4) **K is the point of intersection of (MP) and (AI). (OK) cuts (AM) in N.**
   Show that (ON) is perpendicular to (AM). **(1pt)**