ليسه دي زار

التجربة الرابعة لعام 2009 - 2010 الشهادة المتوسطة

مسابقة في الرياضيات الإنكليزي	المدّة : ساعتان	الإسم :	الرقم :

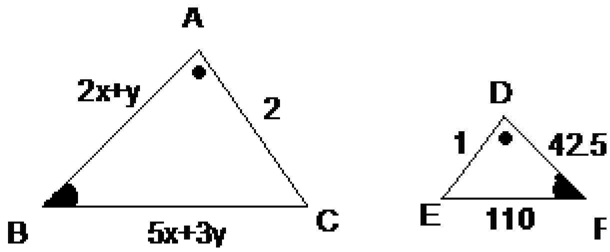
إرشادات عامة:

- يسمح بإستعمال ألة حاسبة غير قابلة للبرمجة
- يمكن الإجابة على ألمسائل بالترتيب االذي تريد
- يرجى الإجابة بخط واضح ومرتب
- العلامة القصوى من 30
- عدد المسائل: 5
-

1st exercise: (6 pts)

In this exercise, the six parts are independent.

1) Given the real number $a=\sqrt{2+\sqrt{3}}$. Show that $\left(a^2+\frac{1}{a^2}\right)$ is a perfect square integer.
2) Find all negative integers that are solutions of the inequality: $\frac{3x+2}{5}-\frac{2x+1}{3}\leq\frac{x+4}{3}$.
3) Five eighths of the grade 9 students succeeded in the math test. Determine the percentage of the students who failed in the test.
4) Knowing that $\alpha+\beta=90°$, show that: $\sqrt{\cos^2\alpha+\cos^2\beta}-2\sqrt{\sin\alpha-\cos\beta+4}=-3$.
5) Knowing that the two triangles ABC and DEF are similar.

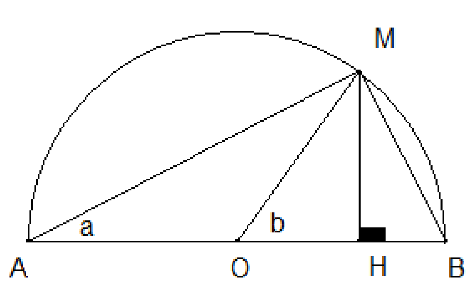


Calculate x and y .

6) Simplify: $\overrightarrow{AB}+\overrightarrow{EC}-\overrightarrow{EB}-\overrightarrow{CA}$.

2nd exercise: (3 ½ pts)

Consider to the right, a semi circle (C) of center O, radius R = 1, and diameter [AB]. M is a point on (C) and H is its orthogonal projection on [AB]. Let $\widehat{MAB}=a$ and $\widehat{MOB}=b$.



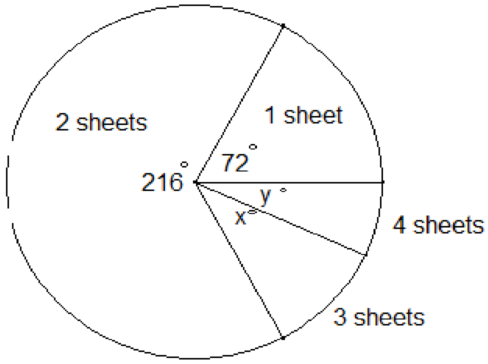
1) Using the two triangles AHM and AMB, calculate cos a. (1pt)
2) a) Determine OH in terms of cos b using a convenient right triangle.(½ pt)

   b) Prove that AH = 1 + cos b. (3/4 pt)

   c) Find a relation between the two angles $\widehat{MAB}$ and $\widehat{MOB}$. Deduce that $2\cos^2 a=1+\cos 2a$. (1pt)
3) Knowing that $\cos 30°=\frac{\sqrt{3}}{2}$, calculate $\cos 15°$ using part 2c. (¾ pt)

3rd exercise: (5 ½ pts)

A surveyor asked 90 students of grade 9 about the number of double sheet papers used in the last math test. The results are represented in the circular diagram to the right.



1) Determine the population, the character and its nature. (½ pt)
2) Knowing that 10 students used 3 double sheet papers, calculate x, and deduce y, then interpret the meaning of y.(1½ pts)
3) Organise the given information in a statistical table that shows the frequencies. (1pt)
4) Calculate the increasing cumulative frequency of the value 3 sheets and interpret its meaning. (1pt)
5) a) Calculate the average number $\overline{x}$ of double sheet papers used. (¾ pt)
   b) The teacher supposes that the number of double sheet papers used by all the students will double in the final exam. Calculate the new mean $\overline{y}$ in terms of the old mean $\overline{x}$. (¾ pt)

4th exercise: (9 pts)

In the plane of an orthonormal system of axes x'ox, y'oy, consider the straight-line (d) of equation $y - 2x = 4$ and the points E(-1 ; -4), H(a – 1; 2), K(2 ; - 4), and S(- 4 ; 2).

1) Draw (d) and plot the points S, E, and K. (1pt)
2) Calculate a so that H belongs to (d). (¾ pt)
3) Let M and N be the x and y intercepts of (d) respectively.
   a) Determine by calculation the coordinates of M and N. (1pt)
   b) Show that $\overrightarrow{MN} = 2\overrightarrow{MH}$, then deduce the position of H with respect to [MN]. (1pt)
4) a) Calculate the slope of (EH) and interpret the result graphically. Deduce the equation of (EH).(1pt)
   b) Determine an equation of (EK), then deduce the nature of triangle EHK. (1 ½ pt)
5) Prove that S is the image of E by the translation of vector $\overrightarrow{KH}$. Deduce the nature of quadrilateral SHKE and prove that its area is 18 unit of area. (2pts)
6) Calculate to the nearest 10^{-2} degree the acute angle that (d) makes with x'ox. (¾ pt)

5th exercise: (6 pts)

Consider the following information:

- C(0 , 3cm) , [AB] diameter ;
- (D) is tangent to (C) at A
- V is a variable point on (C);
- (BV) cuts (D) in N;
- K is the midpoint of [BV]

1) Draw a figure. (¾ pt)
2) Prove by two different ways that (OK) is perpendicular to (BV). (1 ½ pts)
3) a) Prove that the two triangles OBK and VAN are similar. (¾ pt)
   b) Deduce that $NV \times VK = 2 \times OK^2$. (¾ pt)
4) Show that the four points O, A, N, and K belong to the same circle (C') whose diameter is to be determined. (¾ pt)
5) Let I be the center of (C'). On which line does the point I vary when V describes the circle (C)?(¾pt)
6) Let E be the image of K by the vector translation $\overrightarrow{OK}$.
   a) Locate E. (¼ pt)
   b) Show that $\overrightarrow{AE} = \overrightarrow{AV} + \overrightarrow{AO}$. (½ pt)