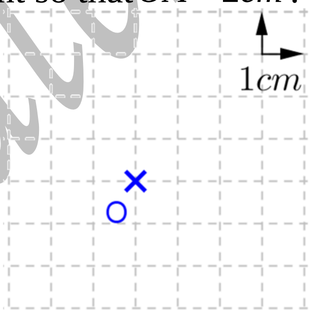
Can you find in the following pictures a fixed point which is at a constant distance from all other points?

Picture			
The fixed point			

Ex-1: In the figure below the point O is fixed where A is any variable point so that $OA = 2cm$.



a. If A is to vary(move) on the plane:

i. Plot three different positions of A.
ii. What do you imagine the path (Locus) of A be? ……………….
iii. What conditions are needed to obtain a circle?
……………………………………………………………………
iv. What does the given $OA = 2cm$ tell you? ……………………..

Def: A circle is a set of variable points that are at a …………..distance from a ………………point in a plane, where the constant distance is called the ………….. and the fixed point is called the…………..

Recall: which segment represents the distance between the point C and the straight line (d)? ………………….
Describe the distance between a point & a straight line.
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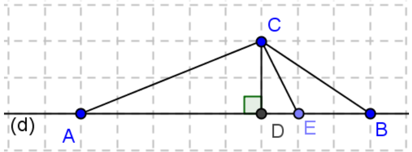
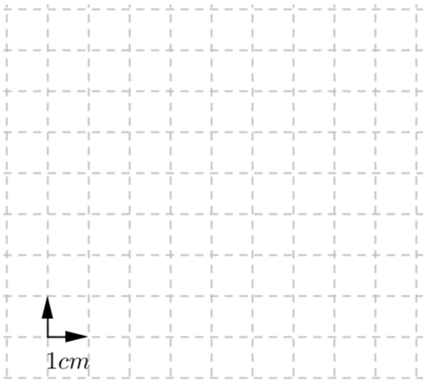


Relative positions of a straight line and a circle

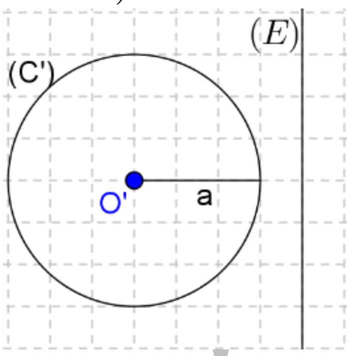
Ex-2: Consider the circle $C(O; r = 1.5cm)$.



a. What does this notation, $C(O;1.5cm)$, tell you?
………………………………………………………
b. Draw (C) on the adjacent grid & trace the straight line (S) such that the distance, d, between (S) and O is $1cm$.
c. Compare d & r : ……………………………………..
d. Does (S), intersect (C)? ……………If yes then in how many points? …..…………………………………
e. Deduce the relative positions of (S) and (C). ………………………..............................
f. Bound d in a way that (S) remains secant to (C)……………………………….......
g. What happens to the length of the chord formed by the points of intersection of (S) & (C), if $d = 0$ ...…………………………………………………………………
h. What do we call the chord that passes through the center of the circle? ………………..

Conclusion: A straight line is secant to a circle if and only if ……………………..

Ex-3: Consider the straight line (E) that is exterior to circle $C'(O';r'=1.5cm)$.



a. Specify the scale used on the grid.

……………………………………………………………………

b. Let H be the orthogonal projection of O' on (E).

i. What does the measure $O'H$ represent? Justify.

……………………………………………………………………

……………………………………………………………………

ii. Compare $O'H$ & r' :……………………………………………

c. What relation should be satisfied so that a straight line is exterior to the circle: ……………………………………………

Conclusion: A straight line is exterior to a circle if and only if ……………………………………………………………………………

Ex-4: Let A be a point on a circle (λ) of center P, and B is the symmetric of A with respect to P so that, $AB = 14 - 4\sqrt{12}\ cm$. And (T) be a straight line that is $d = \dfrac{1}{7+\sqrt{48}}\ cm$ away from P.

a. What does: ***i.*** $[AB]$ represent? ……………………………………………

……………………………………………

ii. d, represent? ……………………………………………

a. Simplify AB & d.

……………………………………………………………………

……………………………………………………………………

……………………………………………………………………

b. What do you notice? ……………………………………………

c. What do you conclude about the relative positions of (T) & (λ) ?

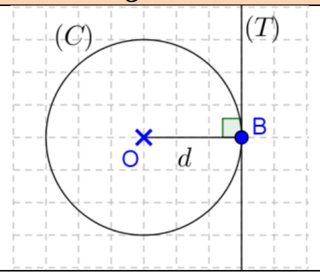
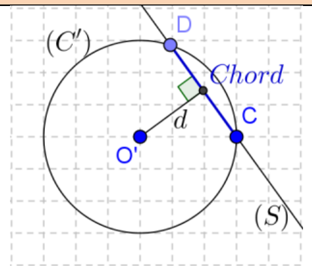
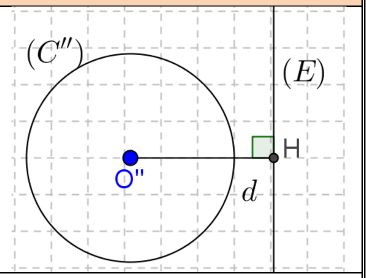
……………………………………………………………………………………………

……………………………………………………………………………………………

d. Write an equation which indicates that a straight line is tangent to a circle.

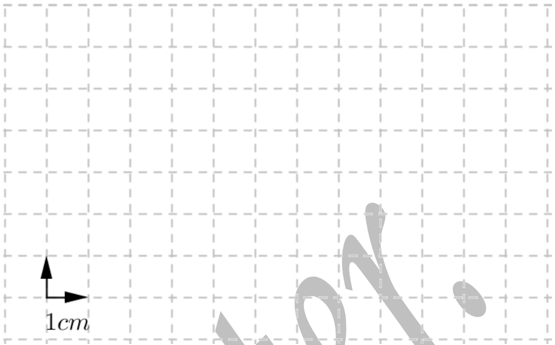
……………………………………………………………………………………………

Conclusion: A straight line is tangent to a circle if and only if ……………………………………………………………………………

Summary			
Representations	*Relative positions between a straight line and a circle*		
	Tangent line	*Secant line*	*Exterior line*
Graphical			
Analytical	$r = d$	$r > d$	$r < d$

Ex-5: Consider the circles $\eta(O,3cm)$ & $\delta(O',2cm)$ where $OO' = \dfrac{2^3+32}{2^3}cm$

1) Prove that OO' is a natural number to be determined.
   …………………………………………………………………
2) Find the sum of the two radii: ……………………….
3) Compare the obtained sum with OO': ……………...
4) Draw on the adjacent grid (η) & (δ).
5) At how many points do (η) & (δ) intersect? ………
6) Deduce the relative positions of (η) & (δ).
   …………………………………………………………………

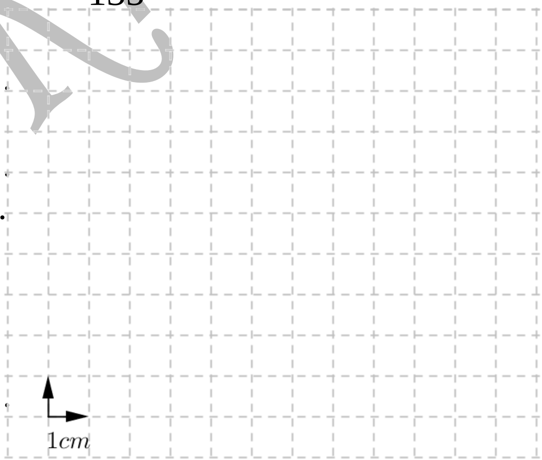


7) When two circles are tangent externally?

Conclusion: Two circles are tangent externally if and only if ……………………………………………………………………

Ex-6: Consider the circles $\lambda(O,5cm)$ & $\Delta(O',3cm)$ where $OO' = \dfrac{3^3+243}{135}cm$

a) Prove that OO' is a natural number to be determined.
   …………………………………………………………………
b) Find the difference between the two radii.
   …………………………………………………………………
c) Compare the obtained difference with OO': ……………
d) Draw on the adjacent grid (λ) & (Δ).
e) At how many points do (λ) & (Δ) intersect? ………….
f) Deduce the relative positions of (λ) & (Δ).
   …………………………………………………………………



g) When two circles are tangent internally?

Conclusion: Two circles are tangent internally if and only if ……………………………………………………………………

Ex-7: Consider the circles $\lambda(O,3cm)$ & $\Delta(O',2cm)$ where $OO' = 0.6 + 2 \times 0.32 \times 10cm$

1) Prove that OO' is a natural number to be determined.
   …………………………………………………………………
2) Find the sum of the two radii………………………….
3) Compare the obtained sum with OO': ………………..
4) Draw on the adjacent grid (λ) & (Δ).
5) At how many points do (λ) & (Δ) intersect? ………….
6) Deduce the relative positions of (λ) & (Δ).
   …………………………………………………………………

7) When two circles are disjoint externally?

Conclusion: Two circles are disjoint externally if and only if ……………………………………………………………………

Ex-8: In the adjacent figure (C) is a circle of center O and radius $r = 5cm$.

1) Trace a circle $C'(O', r' = 3cm)$, so that $OO' = 5.6 - 2\times 0.23\times 10cm$

……………………………………………………………

……………………………………………………………

2) What is the relative position of (C) & (C')?

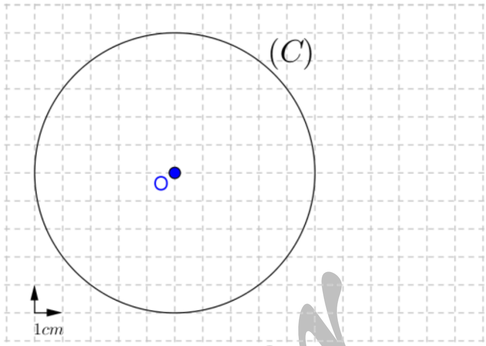
……………………………………………………………

3) Compare $r - r'$ with OO': ……………………………..

……………………………………………………………

4) When two circles are disjoint internally?

……………………………………………………………



Conclusion:	Two circles are disjoint internally if and only if ……………………………………………………………………

Ex-9: Consider the points circles A, B & C so that that $AB = 6cm$, $AC = 3cm$ & $BC = 4cm$

1- Are the given points collinear? Justify.

……………………………………………………………

2- Place the point C.

3- Trace the circles $\lambda(A, AC)$ & $\Delta(B, BC)$.

4- Determine the relative position of the circles (λ) & (Δ).

……………………………………………………………

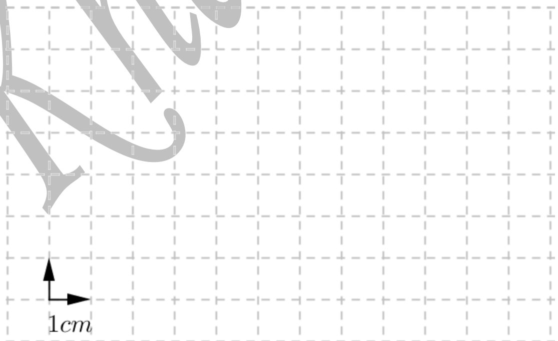
5- Find the sum of the two radii……………………………

6- Frame AB between two numbers to be determined.

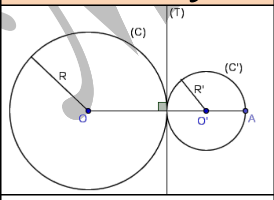
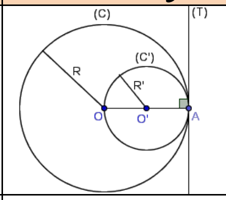
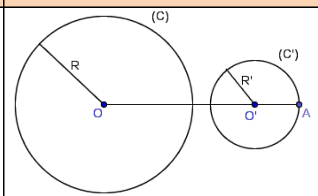
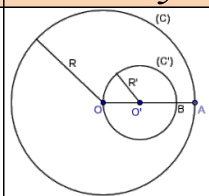
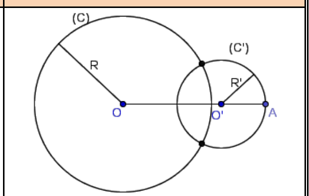
……………………………………………………………

……………………………………………………………

……………………………………………………………



Conclusion:	Two circles are secant (intersecting) if and only if ……………………………………………………………………

Summary					
	Relative positions between two circles				
	Tangent externally	*Tangent internally*	*Disjoint externally*	*Disjoint internally*	*Intersecting*
Graphs					
Relations	$OO' = R + R'$.	$OO' = R - R'$	$OO' > R + R'$.	$OO' < R - R'$	$R - R' < OO' < R + R'$