Al- Mahdi High	*Mathematics*	*11^{th}-Grade*
Name:	*Numerical Sequence*	*A.S-10.*

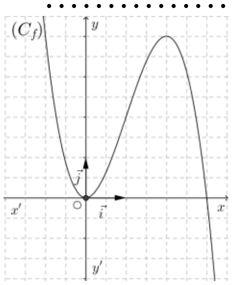
A- *Focusing event:*

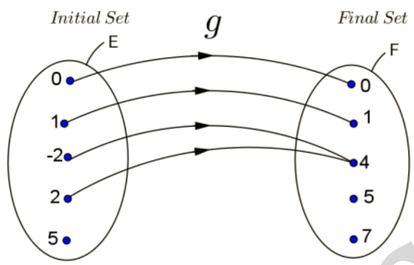
Determine:	- The hundredth odd number: …………………………………………
	- The sum of the first 100 numbers: …………………………………………

B- *Reviewing the definition of a function:*

1) Which of the following representations is a function? Justify.

…………………………………………………………………………………………





h	x	2	5	7	9
	$h(x)$	-3	0	2	4

2) In the above representations:

a. What do x & y determine?

x:…………………………… y:……………………………

b. Determine the domain and range of the above representations:

$D_f =$………………… $D_g =$………………… $D_h =$…………………

$R_f =$………………… $R_f =$………………… $R_h =$…………………

C- *Defining a sequence:*

Consider the sequences of numbers: $(u_n): 0,1,2,3,4,5,6...$; $(v_n): -1.5,-1-0.5,0,0.5,1...$

1- Complete the following tables:

(u_n)	Term	1	2	6
	Term value			

(v_n)	Term	1	3	4
	Term value			

2- Can the 1st term of any sequence take more than one value? ……………………..

3- Is a sequence a function? Justify. ……………………………………………

4- Determine the domain and range of the sequences (u_n)&(v_n):

Domain: ……………………………….. Range: ……………………………

5- Fill in the blanks with the most convenient word: (natural, integer, real, decimal)

- Order of terms of a sequence are: ……………………….numbers.
- Term values of a sequence are: …………………numbers.

6- Are the sets $A=\{-2,1,4\}$ & $B=\{1,4,-2\}$ equal? Justify. …………………………………

7- Are the sequences: $(u_n): 2,4,8,16,32$ & $(v_n): 32,16,8,4,2$ equal? Justify.

…………………………………………………………………………………………

Def$_1$:	- A sequence is a ……………………..from …………….. to ………………
	- A sequence is a list of numbers that follow a certain ………………………..

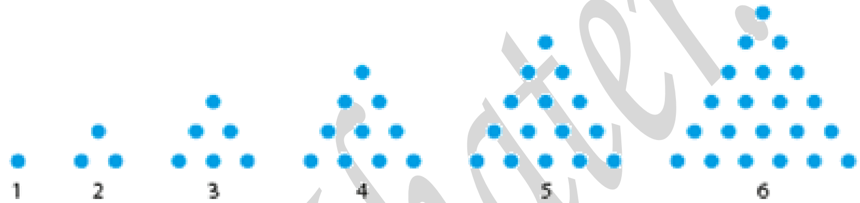
D- Representation of a sequence:

A sequence (U_n) can be represented in two main forms:

Ex1: Find the first three terms and the 50th term of the sequence (b_n) defined by its general term: $b_n = 2 - \frac{1}{n^2}$

…………………………………………………….. ……………………………………………………….
…………………………………………………….. ……………………………………………………….

Ex2: Consider the following pattern:



1) Use the above pattern to complete the following table:

n	1	2	3			
a_n	1	3				

2) What does a_n represent in this pattern? ……………………………………………………

3) Find a relation between each two consecutive terms of the given pattern:

- $a_2 = a_1 + 2$
- $a_3 =$ ....
- $a_4 =$ ....
- $a_5 =$ ....
- $a_6 =$ ....

4) Deduce a general relation between any two consecutive terms $(a_n \;\&\; a_{n+1})$ of the above pattern: ……………………………………………………………………………………

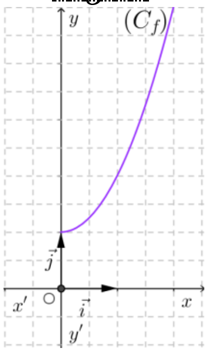
5) Can you find a_{1000} with the above relation? ……………………………………………

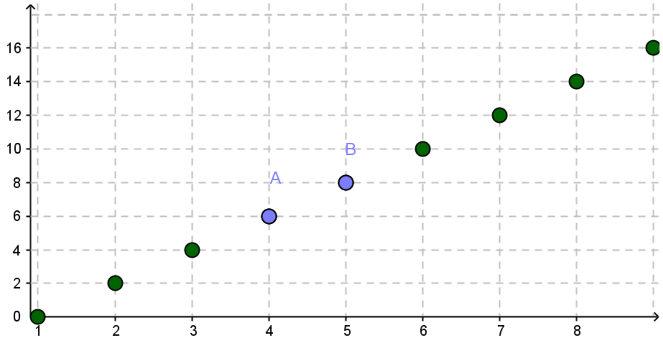
Forms:

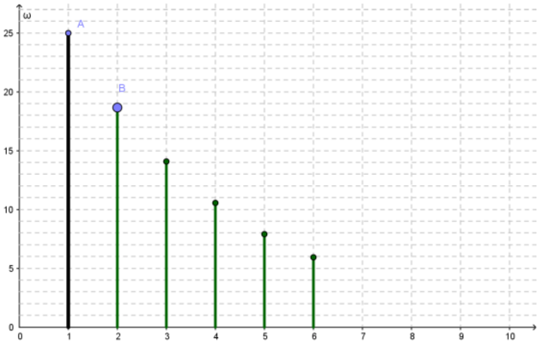
- ***Explicit form:*** A sequence $(u_n): u_n = 3n + 2, \forall n \in \mathbb{N}$, is in explicit form, if its …….….… term is expressed in terms of ………………, and it is useful to find any ……….. of the sequence.
- ***Implicit*** or ***Recursive*** form: A sequence eg: $(u_n): \begin{cases} u_1 = 3 \\ u_{n+1} = 5u_n - 7 \end{cases}$, $\forall$ …., is in implicit or recursive form, if its formula is expressed in terms of two of its ……………………., where the value of the ………………..is given.

E- *Sense of variation of a sequence:*

- *Figure*-1: The curve, C_f, is the representative curve of the function f.
- *Figure*-2: is the graphical representation of the first eight terms of a linear sequence.
- *Figure*-3: is the graphical representation of the first six terms of a geometric sequence.







1) Determine the sense of variation of the:

Function f	Linear sequence	Geometric sequence

***Conclusions*:**	A sequence (u_n) defined by its consecutive terms u_n & u_{n+1} is: - Strictly increasing iff: $u_{n+1} - u_n < 0$ - Increasing iff: …………………………….. - Strictly decreasing iff: …………………….. - Decreasing iff: ……………………………..

Ex: Study the variation of the:

Sequences for all $n \in \mathbb{N}$	*Your solution*
$(u_n): u_n = 3n + 1$	
$(v_n): v_n = \dfrac{1-2n}{n+1}$	
$(w_n): \begin{cases} w_1 = 4 \\ w_{n+1} = w_n - 2 \end{cases}$	

F- Particular sequences:

☆ Arithmetic sequence:

- **Focusing event:** A car company X issues each year a new version, in its first four years the numeration took this form: X_3, X_7, X_{11}, X_{15}..... Determine the serial number that the car will take in the companies' 20th anniversary.

…………………………………………………………………………………………………...

- ***Definition & determination of the general term:***

I- Consider the following arithmetic sequences (a_n) & (b_n) defined by their terms:

The sequence	Terms
(a_n)	$2,4,6,8,10,12...$
(b_n)	$10,7,4,1,-2,-5...$

a- Determine the following differences for the A.S:

- $(a_n):\begin{cases} a_2 - a_1 = ...; & a_3 - a_2 = ... \\ a_4 - a_3 = ...; & a_5 - a_4 = ... \end{cases}$
- $(b_n):\begin{cases} b_2 - b_1 = ...; & b_3 - b_2 = ... \\ b_4 - b_3 = ...; & b_5 - b_4 = ... \end{cases}$

b- What do you notice? ……………………………………………………………..

c- Let d be the common difference, and deduce the definition of an arithmetic sequence:

Def₂:	A sequence (u_n) is ***arithmetic*** if and only if the …………………………………… between any two consecutive terms is ………………..

d- Can you determine the sense of variation of both sequences (a_n) & (b_n), without knowing their general terms? Explain.

……………………………………………………………………………………………………

II- Consider the arithmetic sequence (a_n) of common difference d

a. Complete the following table to find the general term:

Starting from the term: a_0	Starting from the term: a_1
$a_1 = a_0 + d$ $a_2 =$…………………… $a_3 =$…………………… $a_4 =$…………………… . . .	$a_2 = a_1 + d$ $a_3 =$…………………… $a_4 =$…………………… $a_5 =$…………………… . . .
$a_n = a_0 +$………………	$a_n = a_1 +$………………

b. If the first term of the sequence is a_p, then find the general term a_n as a function of a_p:

……………………………………………………………………………………………

c. Now try to answer the focusing event question: ……………………………………

- *Properties of an arithmetic sequence:*

III- Consider the arithmetic sequence (a_n) defined by its terms $(a,b,c,d \,\&\, e) \equiv (3,5,7,9 \,\&\, 11)$.

a. Find the following:

Term	As a function of
b	$a \,\&\, c$:
c	$b \,\&\, d$:
c	$a \,\&\, e$:

b. What do you notice? ……………………………………………………………………

c. Complete the following conclusion:

Conclusions:	The double of any term in …………………….. sequence is equal to ……………………………………………………………………….this property is known as the arithmetic mean.

- *Sum of terms of an arithmetic sequence:*
- *Focusing event:* Determine the sum of the first:

Ten natural numbers	Hundred non-zero natural numbers
0 1 2 3 4 5	1 2 3 . . .

IV- Consider the arithmetic sequence (u_n) of common difference d

a) Complete the following table to find the general term:

Starting from the term: u_0		Starting from the term: u_1	
Term	Value	Term	Value
1 2 3 4 . . .	u_0 $u_1 = u_0 + d$ $u_2 = u_0 + 2d$ $u_3 = u_0 + 3d$ . . .	1 2 3 4 . . .	u_1 $u_2 = u_1 + d$ $u_3 = u_1 + 2d$ $u_4 = u_1 + 3d$ . . .
Sum: $S_n =$		Sum: $S_n =$	

b) Now try to answer the focusing event question: ……………………………………

☆ Geometric sequence:

- *Definition and determination of the general term:*

A) My father & my mother are my first degree ancestors. I take their number to be $A_1 : A_1 = 2$. Each one of my parents had a father and a mother who are my second degree ancestors. I take their number to be $A_2 : A_2 = 4$.

Denote by A_3 & A_4 the third and fourth degree of ancestors

a) Complete the following table:

n	1	2	3	4	5
A_n	$A_1 = 2$	$A_2 = 4$			

b) Determine the ratios: $\frac{A_2}{A_1} = ...; \frac{A_3}{A_2} = ... \frac{A_4}{A_3} = ...$

c) Compare the formed ratios: ……………………………………..

d) Deduce the value of A_5.

e) Verify that: $A_3 = A_1 \cdot r^{3-1}$, where r is the common ratio. ……………………………
………………………………..…………………………………………………………..

f) Use, A_3 to find

i. A_6:…………………………………………………..

ii. A_9:…………………………………………………..

g) Determine a general rule that relates any two none-consecutive terms: ………………..

V- Consider the following geometric sequences (v_n) & (w_n) defined by their terms:

The sequence	Terms
(v_n)	2,4,8,16,32,64...
(w_n)	$81,27,9,3,1,\frac{1}{3},...$

h) Determine the following ratios for the G.S:

- $(v_n): \begin{cases} \frac{v_2}{v_1} = ...; & \frac{v_3}{v_2} = ... \\ \frac{v_4}{v_3} = ...; & \frac{v_5}{v_4} = ... \end{cases}$ $(w_n): \begin{cases} \frac{w_2}{w_1} = ...; & \frac{w_3}{w_2} = ... \\ \frac{w_4}{w_3} = ...; & \frac{w_5}{w_4} = ... \end{cases}$

i) What do you notice? ……………………………………………………..

j) Let r be the common ratio, and deduce the definition of the geometric sequence:

Def₂:	A sequence (u_n) is ***geometric*** if and only if the ………………………………… between any two consecutive terms is ………………..

e- Can you determine the sense of variation of both sequences (v_n) & (w_n), without knowing their general terms? Explain.

……………………………………………………………………………………………

VI- Consider the geometric sequence (a_n) of common ratio r

a. Complete the following table to find the general term:

Starting from the term: a_0	Starting from the term: a_1
$a_1 = a_0 \cdot r$ $a_2 =$ .... $a_3 =$ .... $a_4 =$	$a_2 = a_1 \cdot r$ $a_3 =$ .... $a_4 =$ .... $a_5 =$ ....
$a_n = a_0 \cdot r^{\cdots\cdots}$	$a_n = a_1 \cdot r^{\cdots\cdots}$

b. If the first term of the sequence is a_p, then find the general term a_n as a function of a_p:

....

Def₂: A sequence (u_n) is ***geometric*** if and only if the ratio, r, between any two consecutive terms is ……………, where the general term $u_n = u_1 \cdot r^{\cdots\cdots}$ or $u_n = u_p \cdot r^{\cdots\cdots}$ such that n & p belong to $\mathbb{N}$.

- *Sum of terms of a geometic sequence:*

VII- Consider the sum, $S_n = \sum_{i=0}^{n} u_n$ of n terms of a G.S (u_n) with common ratio $r \neq 1$:

a. Write $S_n = \sum_{i=0}^{n} u_n$ in expanded form: ....

b. Find the product, $r \cdot S_n$: ....

c. Deduce the value of S_n in terms of r, u_1 & u_n exclusively:

d. Determine S_n in terms of the first term and the common ratio of the sequence.

....

☆ The sum of n terms of a G.S (u_n) with a common ratio $r \neq 1$ starting from:

➯ u_0 is given by: $\sum_{i=0}^{n} u_n = u_0 + u_1 + u_2 + \ldots + u_n = S_n = \frac{u_0\left(1 - r^{n+1}\right)}{1 - r}$

➯ u_1 is given by: $\sum_{i=1}^{n} u_n = u_1 + u_2 + \ldots + u_n = S_n = \frac{u_1\left(1 - r^{n}\right)}{1 - r}$