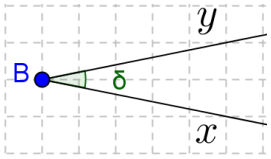
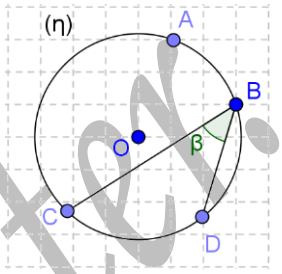
1) What does the adjacent figure represents?
……..



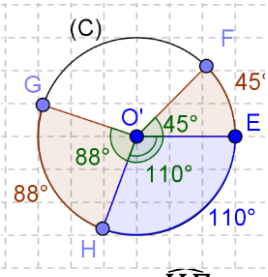
2) Name its elements: ……..
3) Let A, B, C & D be any four points on a circle $\eta(O, r\,cm)$:
   a. Name 5 angles whose vertices are the center of the circle.
   ……..
   b. What name can you impose for such angles?
      i) Acute angle.
      ii) Central angle.
      iii) Exterior angle
   c. Is β a central angle? Justify.
   ……..



Def: A central angle is ……
……...

ξ *Properties of central angles:*

I) Observe the adjacent figure and answer the following questions:

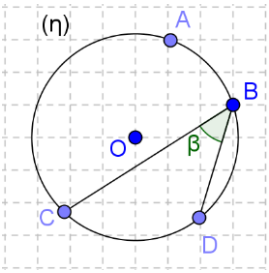


a) What is the measure of the arcs?
$mes\widehat{EF}$…….. $mes\widehat{GH}$……..
b) Name the arc intercepted (facing) by the angles:
$G\hat{O}'H$…….. $E\hat{O}'F$……..
c) Compare:
$E\hat{O}'F$…….. $mes\widehat{EF}$ $G\hat{O}'H$……..$mes\widehat{GH}$ $H\hat{O}'E$…….. $mes\widehat{HE}$
d) What do you notice? ……

Conclusion: The measure of a central angle is ……
……...

e) What is the sum of central angles in a circle? ……...
f) Deduce the measure of:
   i. Sum of arcs held by a circle: ……..
   ii. $arcFG$ ……

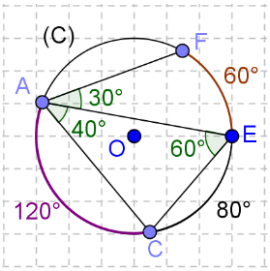
II) Let A, B, C & D be any four points on a circle $\eta(O, r\,cm)$:
   a) Name 5 angles whose vertices are on the circle.
   ……..
   b) What name can you impose for such angles?
      ii) Inscribed angle.
      iii) Central angle.
      iv) Exterior angle



Def: An …… angle is ……
……...

ξ *Properties:*

A) Observe the adjacent figure and answer the following questions:



1. Complete by either of the terms "Major or Minor:
   arcAFE :..... *arcACE* :.....
2. Define:

Major arc	
Minor arc	

3. Complete and compare:

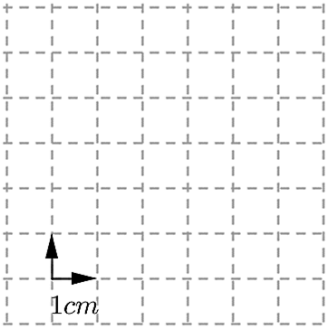
	Inscribed angle	Measure of intercepted arc	Comparison
1)	$E\hat{A}F =$	$mes\widehat{EF} =$	
2)	$A\hat{E}C =$	$mes\widehat{AC} =$	
3)	$C\hat{A}E =$	$mes\widehat{CE} =$	

Conclusion: The measure of an inscribed angle is ……
……..

4. Determine the measure of central angles:
   i) $E\hat{O}F$:……
   ii) $A\hat{O}F$:……
5. What can you say about the angles:
   a) $E\hat{O}C$ & $E\hat{A}C$ ……..
   b) $A\hat{O}C$ & $A\hat{F}C$ ……..
6. Compare the measure of:
   $E\hat{A}F$.....$E\hat{O}F$ $C\hat{A}E$.....$C\hat{O}E$ $A\hat{E}C$.....$A\hat{O}C$

Conclusion: The measure of an inscribed angle is ……
……..

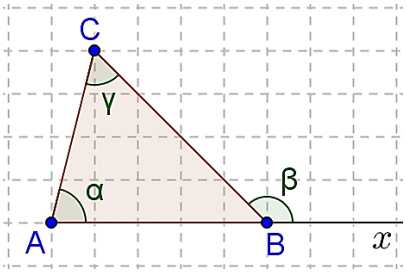
B) Let M be any point on the circle (c) of center O and diameter $[AB]$.



1) Trace the figure.
2) What is the type of angle $A\hat{M}B$? ……
3) Calculate the measure of $A\hat{M}B$.
   ……
   ……
   ……

Conclusion: The measure of an inscribed angle is ……
……..

C) ABC is any triangle where B is on ray $[Ax)$.

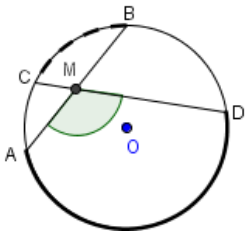


1) Determine the sum of β & $A\hat{B}C$. Justify.
   ……
   ……
2) Deduce the relation between the angles α, β & γ.
   ……
   ……
   Rule: ……

ξ *Interior angles:*

By definition, an interior angle: is an angle enclosed between ***two chords intersecting inside the circle***.

If $[AB]$ *and* $[CD]$ are chords of (c), then prove that: $A\hat{M}D = mes\dfrac{\overset{\frown}{AD}+\overset{\frown}{BC}}{2}$



……

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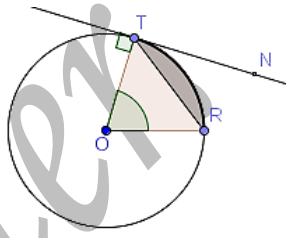
……

……

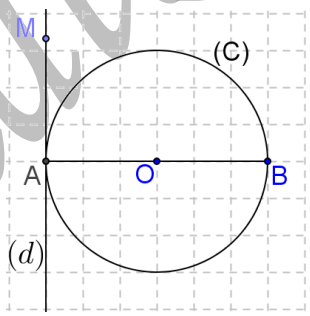
ξ *Angle between a tangent and a chord:*

By definition: Angle enclosed between a ***tangent*** and a ***chord*** issued from the point of tangency is equal to half the intercepted arc.



Hence, $R\hat{T}N = mes\dfrac{\overset{\frown}{TR}}{2}$.

☆ *App:* In the adjacent figure, determine the angle formed between the tangent (d) and the diameter $[AB]$



……

……

……

Tangent theorem:……

ξ *Exterior angles:*

The following angles are exterior:

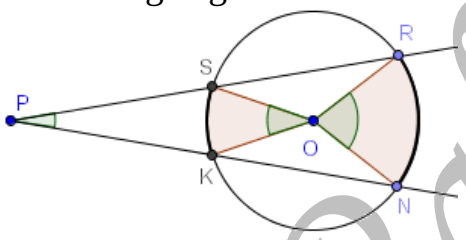


Fig-1

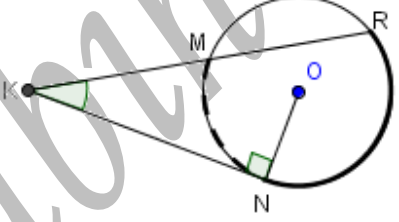


Fig-2

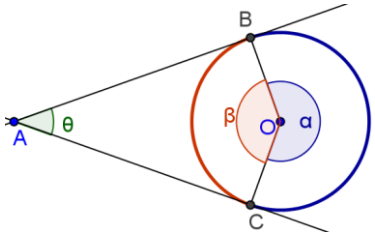


Fig-3

Examine the above angles and deduce the definition of an exterior angle.

Def: An exterior angle is an angle ……

……

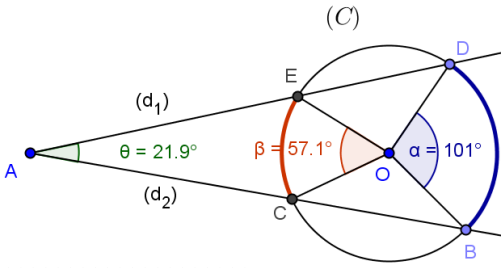
ξ Measure of exterior angle:

a) What do the following angles represent?

✓ α: ……

✓ β: ……

✓ θ: ……



b) Determine the following measurements:

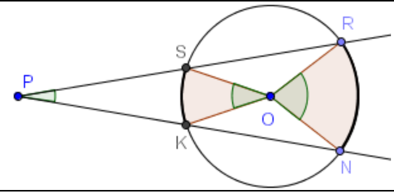
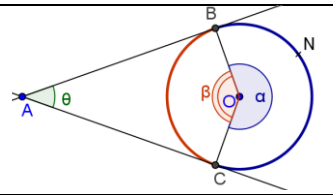
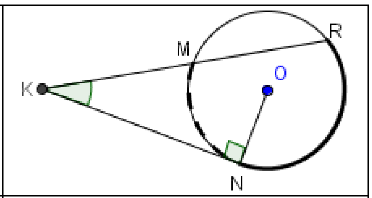
$\alpha - \beta =$…… $2\theta =$……

c) Compare the obtained results: ……

Conclusion: The measure of an exterior angle $= \dfrac{1}{2} mes(\ldots\ldots\ldots\ldots)$

You can press the figure to access GeoGebra and examine more cases

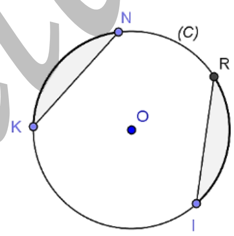
ξ Type of exterior angles:

Graphical form:			
Description: Exterior angle formed between	Two secants	……….......................	……….......................
Measure:	$R\hat{P}N = mes\frac{\overset{\frown}{RN}-\overset{\frown}{SK}}{2}$.		

❖ Properties:

➢ ***Chords subtended by equal arcs:***

In the adjacent figure the measure of the arcs NK & RI are equal.



i) Prove that the chords $[NK]$ & $[RI]$ are equal.

……………………………………………………………………………………………

……………………………………………………………………………………………

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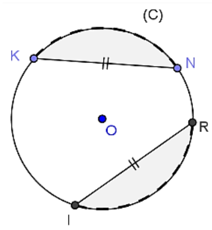
……………………………………………………………………………………………



ii) Is the converse true? Justify.

……………………………………………………………………………………………

……………………………………………………………………………………………

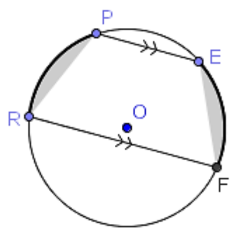
……………………………………………………………………………………………

……………………………………………………………………………………………

Properties:	1- Chords subtended by ……………………………………………are equal. 2- Arcs held by equal ……………………………………………………………

➢ ***Arcs*** included between ***parallel chords*** in a circle:

In the adjacent figure the chords $[PE]$ & $[RF]$ are parallel.



a. Prove that the arcs PR & EF admit equal measures.

……………………………………………………………………………………………

……………………………………………………………………………………………

……………………………………………………………………………………………

b. What can you say about the chords $[PR]$ & $[EF]$?

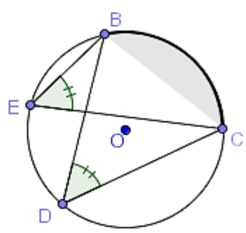
……………………………………………………………………………………………

……………………………………………………………………………………………

Properties:	3- Arcs included between........……………………………………are equal. 4- Chords ………………...................................chords………..…………..

➢ ***Inscribed angles intercepting the same arc are equal.***

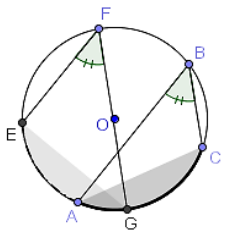
Prove that: $B\hat{E}C = B\hat{D}C$



……………………………………………………………………………………………

……………………………………………………………………………………………

➢ ***Equal angles intercept equal arcs*** and vice versa.

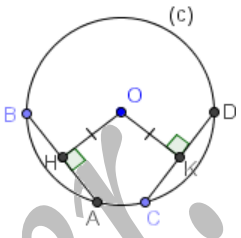


Decoding: If $E\hat{F}G = A\hat{B}C$.

Then, $\overset{\frown}{EG} = \overset{\frown}{AC}$.

➢ ***Chords equidistant from the center of the circle are equal***, and vice versa.

Decoding: If $OH = OK$, ***then*** chord AB = chord CD.

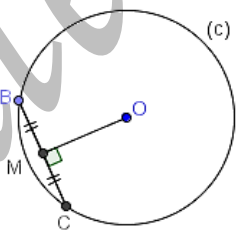


Converse: If chords AB and CD are equal, ***then*** $OH = OK$

Therefore, O is equidistant from points H & K.

➢ ***Straight line joining center and midpoint of a chord:***

Let M be the midpoint of $[EC]$ on the circle (c) of center O.



1) What does $[EC]$ represent with respect to (c)?

……

……

2) What is the nature of traingle EOC ? Justify.

……

……

……

3) What is the relative positions of $[OM]$ and $[BC]$? Justify.

……

……

……

Conclusion:	The straight line joining the center of the circle and the midpoint of a chord is ……

💣 ***Rule-1:*** (Perimeter of a circle): $2\pi R \longrightarrow 360^{o}$ (***Greatest angle in a circle***)

(Length of an arc): $L \longrightarrow R\hat{O}N$ (***Central angle corresponding to L***)

$$\text{Length of an arc} = \frac{2\pi R \times R\hat{O}N}{360^{\circ}}$$

💣 💣 ***Rule-2:*** (Area of a circle): $\pi R^2 \longrightarrow 360^{\circ}$ (***Greatest angle in a circle***)

(Area of a circular sector): A $\longrightarrow$ $R\hat{O}N$ (***Central angle corresponding to A***)

$$\text{Area of a circular sector} = \frac{\pi R^2 \times R\hat{O}N}{360^{\circ}}$$