Al Mahdi High Schools *Mathematics* *11th-Grade*

(Al-Hadath)

Name: . . . . *" Orthognality "* *A.S-11.*

Collinearity in space

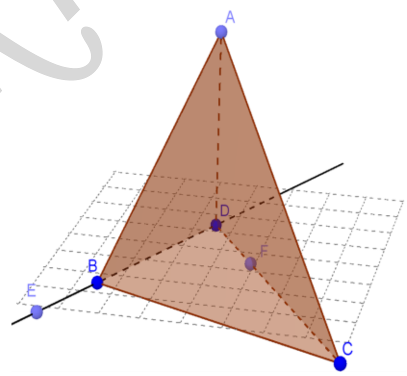
Property: To prove that three distinct points in space are collinear, it is enough to show that these points belong to two distinct planes.

App-1:

In the tetrahedron $ABCD$, let E & F be any two points on (BD)&$]CD[$ respectively, so that (EF)&(BC) intersect at G. Let H be any point on $]AD[$ such that I & K are the respective intersection points of (EH)&(AB) and (FH)&(AC).

1- Determine the straight line $(\Delta)=(ABC)\cap(EFH)$

……………………………………………………………………….
……………………………………………………………………….
……………………………………………………………………….
……………………………………………………………………….
……………………………………………………………………….



2- Deduce that the points G, I & K are collinear.

……………………………………………………………………….
……………………………………………………………………….
……………………………………………………………………….
……………………………………………………………………….

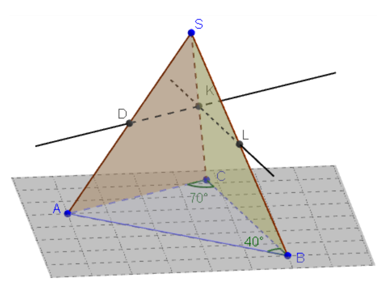
Angle between two lines

App-2:

In the adjacent figure $SABC$ is a tetrahedron, where D, K & L are the respective midpoints of sides $[AS],[CS]$&$[BS]$ and $A\hat{B}C=40^\circ$ & $A\hat{C}B=70^\circ$.

1- What is the relative position of

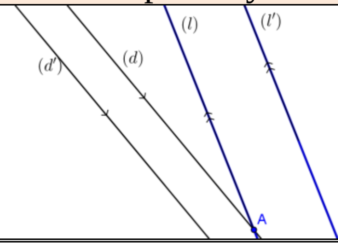
(BC)&(LK)?	……………………………………………………… ………………………………………………………
(KD)&(AC)?	……………………………………………………… ………………………………………………………
(AC)&(DL)?	……………………………………………………… ………………………………………………………



2- Deduce the angle formed between the straight lines:

(KD)&(KL)	……………………………………………………………… ………………………………………………………………
(AC)&(DL)	……………………………………………………………… ………………………………………………………………

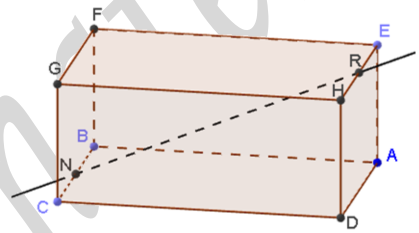
3- What do you conclude?

Analytically	Graphically
To find the angle between any two non-coplanar lines, we determine the angle formed between any …………………… …………………………………………… ……………………………………………	

App-3: Orthogonal lines

Def: Two lines are orthogonal *iff* the included angle is $\frac{\pi}{2}$

Eg:



St.lines	Intersection	Angle	Perpendicular	Orthogonal
$(CD)\&(CB)$	B	$\frac{\pi}{2}$	✓	✓
$(CD)\&(AE)$	$\emptyset$	$\frac{\pi}{2}$		✓

Use the paving stone $ABCDEFGH$ where $R\&N$ are the respective midpoints of $[EH]\&[BC]$ to complete the table

Perpendicular	Justification	Orthogonal	Justification
$(AB)\perp$		$(AB)\perp\!\!\!\!\perp$	
$(AB)\perp$		$(AB)\perp\!\!\!\!\perp$	
$(AB)\perp$		$(AB)\perp\!\!\!\!\perp$	
$(NR)\perp$		$(NR)\perp\!\!\!\!\perp$	
$(NR)\perp$		$(NR)\perp\!\!\!\!\perp$	

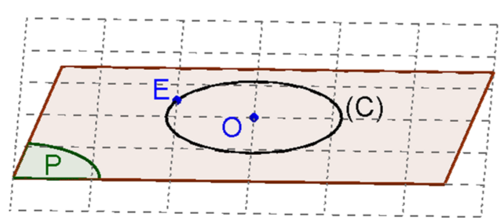
Conclusions: Two straight lines are:

- Orthogonal if: ……………………………………………………………………
- Perpendicular if: ……………………………………………………………………

Note that: Two orthogonal lines are not necessarily intersecting.

Two perpendicular lines are always intersecting.

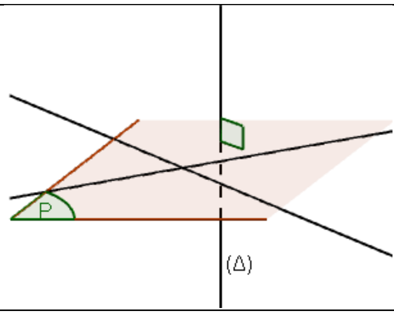
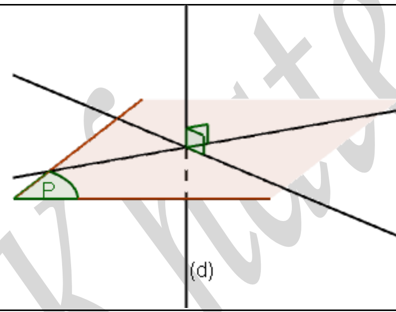
Ex: In the adjacent figure, $C(O,OE)$ is in plane (P). Trace (RN) the perpendicular bisector of $[OE]$ in (P) & (Δ) the perpendicular to (P) at E.



1) Complete figure.
2) Prove that (RN) is orthogonal to (FO), where $F\in(\Delta)$

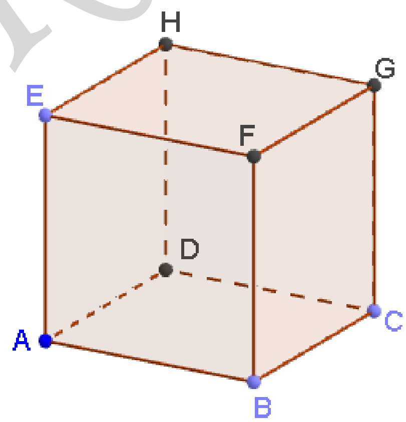
……………………………………………………………………..
……………………………………………………………………..
……………………………………………………………………..

Line orthogonal to a plane in space

	Definition	Proof
Analytically	A line is perpendicular to a plane if it is orthogonal to every line subset of this plane.	To prove that a line is perpendicular to a plane, it is enough to show that this line is orthogonal to two intersecting lines subset of this plane.
Geometrically		

App-3:

Consider the cube $ABCDEFGH$.



1) Show that:
   a. (AE) is perpendicular to (BCD) .

   ……………………………………………………………..
   ……………………………………………………………..
   ……………………………………………………………..

   b. (GH) is perpendicular to (ADE) .

   ……………………………………………………………..
   ……………………………………………………………..
   ……………………………………………………………..

2) Deduce that:
   i. $(AE)\perp(BD)$.

   ……………………………………………………………..
   ……………………………………………………………..

   ii. $(GH)\perp(ED)$.

   ……………………………………………………………..
   ……………………………………………………………..

3) Prove that (AE) is perpendicular to (FGH) .

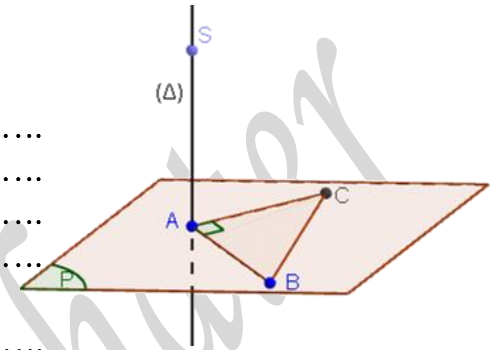
   ……………………………………………………………..
   ……………………………………………………………..

4) What is the relative positions of the planes: (BCD) & (FGH) .

   ……………………………………………………………..
   ……………………………………………………………..
   ……………………………………………………………..

Properties

Consider ABC to be a right isosceles triangle at A in a plane (P). Let S be a point on straight line (Δ) the perpendicular to (P) at A.



1) Prove that (AB) is perpendicular to (SAC)

……………………………………………………………………..
……………………………………………………………………..
……………………………………………………………………..
……………………………………………………………………..

2) Show that the triangles SAB & SAC are congruent.

……………………………………………………………………..
……………………………………………………………………..
……………………………………………………………………..
……………………………………………………………………..

3) Deduce the nature of the triangle SBC .

……………………………………………………………………..
……………………………………………………………………..
……………………………………………………………………..
……………………………………………………………………..

Prop-1	If a point S belongs to (Δ), the ***Perpendicular*** to (P) at A, where A is equidistant from B & C in (P) then S is equidistant from B & C and vice versa.

4) Let I be the midpoint of $[BC]$. Show that (BC) is perpendicular to the plane (SAI)

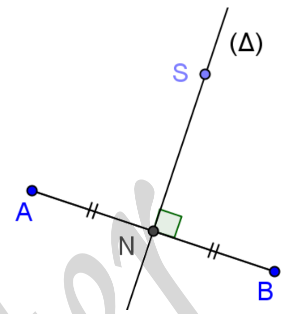
……………………………………………………………………..
……………………………………………………………………..
……………………………………………………………………..
……………………………………………………………………..

5) Plot N the orthogonal projection of A on (SI). Prove that (AN) is orthogonal to (SB).

……………………………………………………………………..
……………………………………………………………………..
……………………………………………………………………..
……………………………………………………………………..

Mediator plane of a segment

<ins>REMINDER</ins>:



1) What does (Δ) represent in the adjacent figure?

…………………………………………………………………..

2) Describe S with respect to A & B. Justify.

…………………………………………………………………..

…………………………………………………………………..

Def:	A mediator plane is a plane in which every point in it is equidistant from two extremities of a given segment.

App-4:

Consider the regular tetrahedron $SABC$, and I the midpoint of $[CD]$.

1) Draw a figure.

2) Prove in two different ways that SBI is a mediator plane of $[CD]$

1^{st} – way	2^{nd} – way

Conclusion	To prove that (P) mediator plane for a segment $[AB]$, it is enough to show that: 1^{st} - way: Three points …………………………………………………… 2^{nd} - way: The plane (P) is: - …………………………………………………… - ……………………………………………………