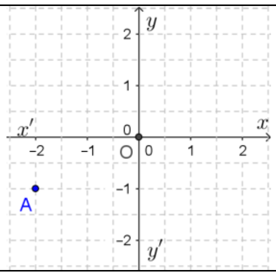
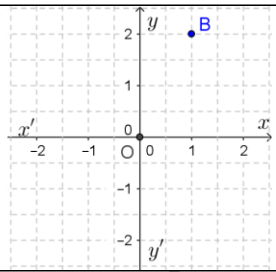
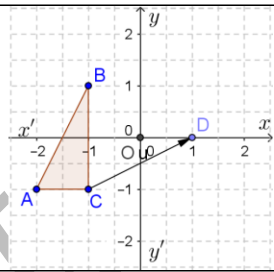
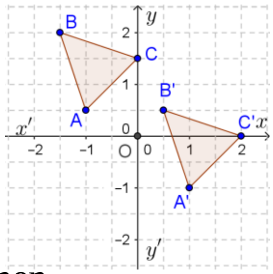

↳ ***Focusing event:*** **Translate** (<u>*slide or move*</u>)the below items following the specified instructions:

| | | | |
|---|---|---|---|
| Figures |  |  |  |
| Instructions | 2 units along $+ve$, $x-axis$ | 2 units along $-ve$, $x-axis$ & 1 unit along $-ve$, $y-axis$ | As the translation from $C$ to $D$ |
| Outcomes | 1) What is the translation (image) of a point? ……………………………………… | Triangle? …………… | |
| | 2) Did translation preserve the proerties of the triangle? ……………………….. | | |
| | 3) What did you need to find the image of $A$ in fig-1? …………………………. | | |



Find the elements needed to translate triangle $ABC$ to $A'B'C'$ ?

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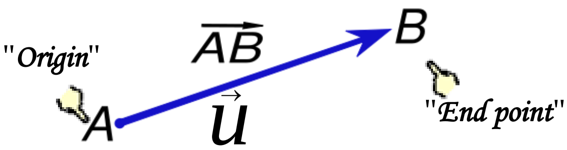
↳ ***Intoducion:*** A **translation** "slides" an object a ***<u>fixed</u>*** distance in a given ***<u>direction</u>***. The original object and its translation have the **same shape and size**, and they **face in the same direction**.

↳ If a point $A$ moves towards another point $B$ which are $5m$ apart and ***back*** then,

- ☆ $A$ moves a total distance of: ……………
- ☆ The displacement of $A$ is: ……………….

↳ ***<u>Definition of a vector</u>:***

A vector $\vec{u}$ or $\vec{AB}$ is an ***oriented*** segment having two ***extremities***, an ***origin*** and an ***end point***.



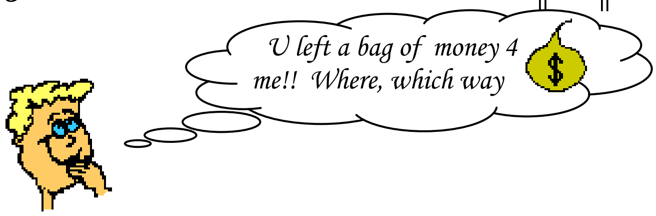

↳ ***<u>Properties of a vector</u>:***

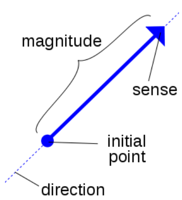
In elementary mathematics, a ***vector*** is a geometric object defined by its properties:

*a) <u>Direction</u>:* the st. line that ***holds*** $\vec{AB}$ or any st. line ***parallel*** to (*AB*); ***e.g.*** vertical, along (*d*)...

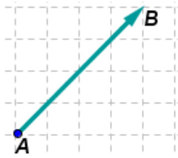
*b) <u>Sense</u>:* orientation from an initial point to a final point; ***e.g.*** left to right or from pt *A* to *B*…

*c) <u>Magnitude</u>:* length (modulus) or norm $\|\vec{AB}\|$ of the given vector.



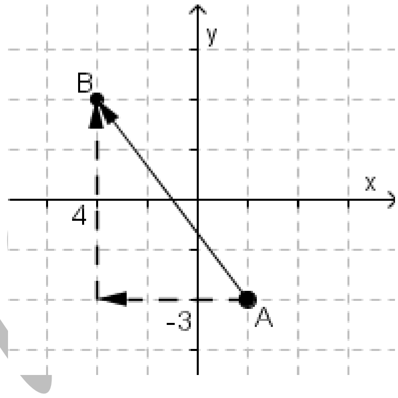

- ***Representation of a vector:*** A vector is frequently represented by a ***segment with a definite direction***, or graphically as an arrow, connecting an ***initial point*** $A$ with a ***terminal point*** $B$, and denoted by: $\overrightarrow{AB}$



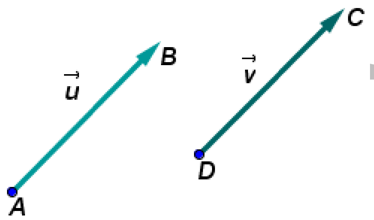

- ***How to locate a point using a vector?***

To locate a point ***B*** the translate of $A(1;-2)$ knowing coordinates of $\vec{AB}(-3;4)$

1. Plot the given point ***A***.
2. If $x_{\vec{AB}} = -3$, then
   **1.** Direction : Parallel to $x$-axis.
   **2.** Sense : Move to the left.
   **3.** Magnitude:3-units.
3. If $y_{\vec{AB}} = +4$, then
   **1.** Direction : Parallel to y-axis.
   **2.** Sense : Move upwards.
   **3.** Magnitude:4-units.
4. Finally, plot point $B$.



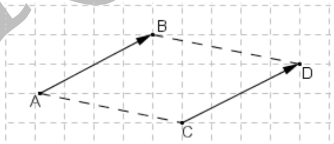

- Special vector:
- ***Equal vectors:***

| ***Analytic approach*** | ***Geometric approach*** | ***Conclusion*** |
|---|---|---|
| Any two vectors $\vec{u}$ *and* $\vec{v}$ are ***equal*** *iff* they admit:<br>- Same ***direction***.<br>- Same ***sense***.<br>- Same ***magnitude***. |  | ***Therefore***, vectors $\vec{u}$ *and* $\vec{v}$ are ***equal***.<br>*And we write:* $\vec{u}=\vec{v}$ *OR* $\vec{AB}=\vec{DC}$ |



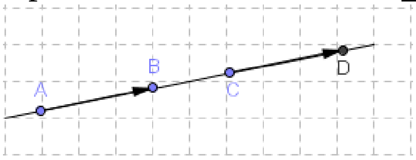
✓ **Significance of equal vectors:**

**a.** If $\vec{AB}=\vec{CD}$, then $C$ is the fourth vertex of the ***parallelogram*** $ABDC$ .



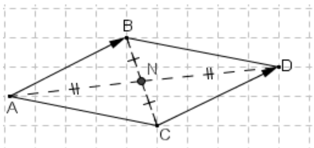

*Conversely:* If $ABDC$ is a ***parm*** then, $\vec{AB}=\vec{CD}$ .

**b.** If $\vec{AB}=\vec{CD}$, then the points $A$, $B$, $C$ and $D$ are ***collinear***.

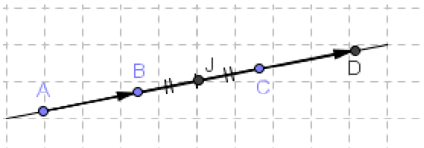



☆ ***Properties of equal vectors:***

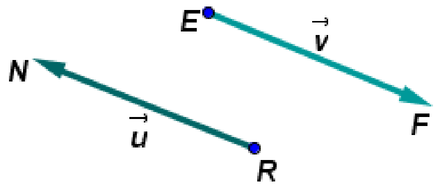
If $\vec{AB}=\vec{CD}$, then the segments $[AD]$ and $[BC]$ have the ***same midpoint***.



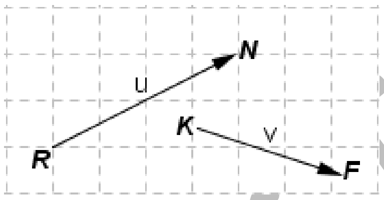
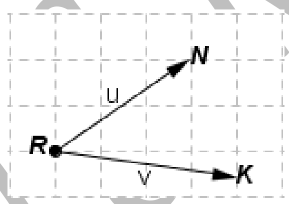
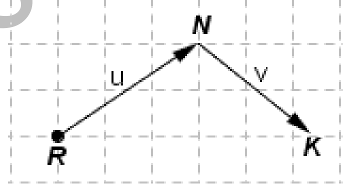

Or



↳ ***Opposite Vectors***: Opposite and equal vectors admit same significance and properties.

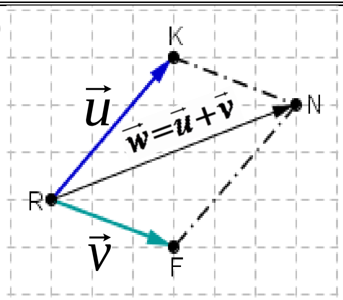
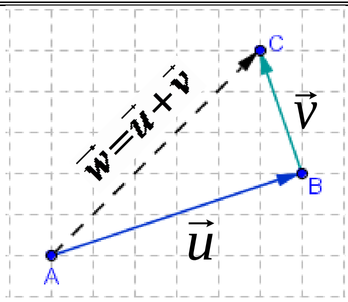
| Analytic approach | Geometric approach | Conclusion |
|---|---|---|
| Any two vectors $\vec{u}$ and $\vec{v}$ are opposite if they have:<br>- Same direction.<br>- Same magnitude.<br>- But opposite senses. |  | ***Therefore***, $\vec{u}$ and $\vec{v}$ are **opposite**.<br>And we write: $\vec{u} = -\vec{v}$<br>OR $\vec{RN} = -\vec{EF}$<br>OR $\vec{RN} + \vec{EF} = \vec{0}$ |

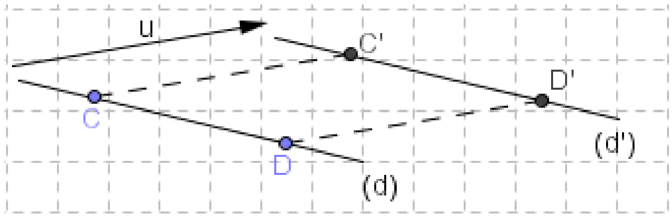
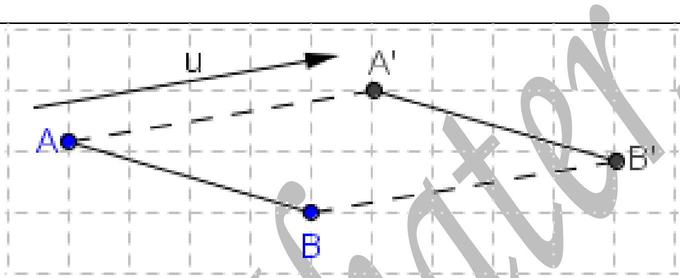
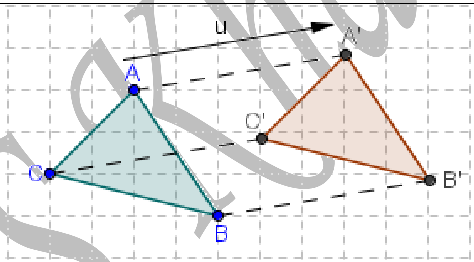
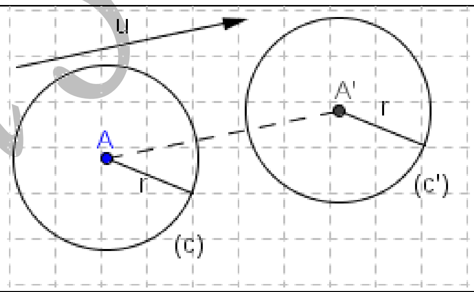
***Types of vectors***:

| | *Free vectors* | *Vectors of the same origin* | *Consecutive Vectors* |
|---|---|---|---|
| **Definition** | Are vector having neither a common origin nor common extremity. | Are vector having a common origin only. | Are vectors having the extremity of the first as the origin of the second. |
| **Geometric approach** |  |  |  |

↳ ***Sum of two vectors***:

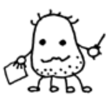
There are two main methods to add two or more vectors having the same ***coefficients***:

| | *1st – Method* | *2nd – Method* |
|---|---|---|
| | ***Parallelogram Rule*** | ***Chasles' Rule*** |
| ***Used*** | *If vectors have the same origin* | *If vectors are consecutive* |
| ***Method*** | *Complete the parallelogram* | *Join the first origin to the last extremity.* |
| *Graphical representation* |  |  |
| *Analytical approach* | The sum of two vectors with same origin; is a vector with same origin and its extremity is the fourth vertex of the parm.<br>$\vec{RF} + \vec{RK} = \vec{R\underbrace{N}_{4thVertex}}$. | The sum of two consecutive vectors; is a vector with orgin of 1st and extremity of the last.<br>$\vec{AB} + \vec{BC} = \vec{AC}$. |

| | *How to find the Image or translation of some geometric figures* | |
|---|---|---|
| *Figures* | *Procedure* | *Graphical representation* |
| *Lines* | *To translate a line, translate any two points on this line.* |  |
| *Segments* | *To translate a segment, translate its extremities.* |  |
| *Triangles* | *To translate a triangle, translate its vertices* |  |
| *Circles* | *To translate a circle, translate its center and keep the same radius* |  |

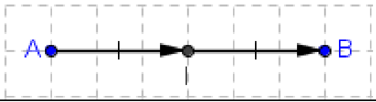
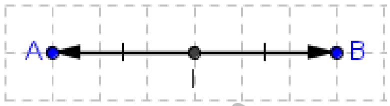
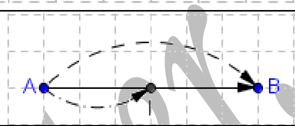
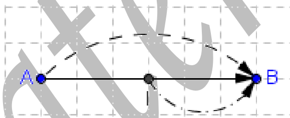
## ➪ ***Properties of translation:***

**Translation preserves**
- Parallalism and orthogonality
- Areas and congruencies
- Lengths and midpoints
- Angles and collinearity



## Midpoints and Vectors :

If $I$ is the ***midpoint*** of [$AB$] then;

| | Analytic approach | Graphical representation |
|---|---|---|
| ☆ | $\vec{AI} = \vec{IB}$. |  |
| ☆ | $\vec{IA} + \vec{IB} = \vec{0}$. |  |
| ☆ | $\vec{AB} = 2\vec{AI}$. |  |
| ☆ | $\vec{AB} = 2\vec{IB}$. |  |

☆ *Conversely:*

*If* $\begin{cases} \vec{AI} = \vec{IB} \\ \vec{IA} + \vec{IB} = \vec{0} \\ \vec{AB} = 2\vec{AI} \\ \vec{AB} = 2\vec{IB} \end{cases}$ *then,* ***I*** is the ***midpoint*** of [***AB***]

✓ Use one of the above vector relations to determine the coordinates of the midpoint.

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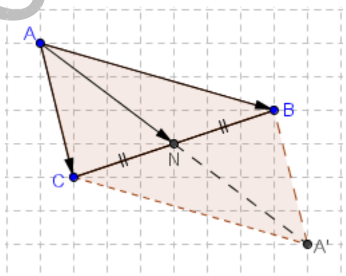
## Medians and Vectors :

If [***AN***] is a median relative to [***BC***] then; $\vec{AB} + \vec{AC} = 2\vec{AN}$.




☆ *Conversely: If* $\vec{AB} + \vec{AC} = 2\vec{AN}$ then, [***AN***] is the median relative [***BC***].

☆ *Genrerally: If* $A$ is any point in the plane and $M$ is the midpoint of $[BC]$ then we write:

$$\vec{AB} + \vec{AC} = 2\vec{AM}.$$

✓ Use Chasle's rule for sum of consecutive vectors to prove the above vector relation.

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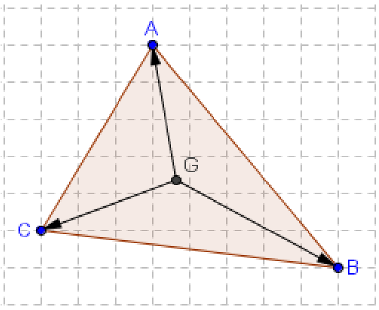
## Centroid and Vectors:

If **G** is the center of gravity (***Centroid***) of triangle ***ABC*** then, $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=\overrightarrow{0}$.




☆ *Conversely:* ***If*** $\overrightarrow{GA}+\overrightarrow{GB}+\overrightarrow{GC}=\overrightarrow{0}$ then; ***G*** is the ***center of gravity*** of triangle ***ABC***.

## Vectors and Coordinate system :

In a reference plane any vector $\overrightarrow{AB}$ has two coordinates:

- To find coordinates of a vector $\overrightarrow{AB}$, use the following relations:

$$X_{\overrightarrow{AB}} = x_B - x_A.$$

$$Y_{\overrightarrow{AB}} = y_B - y_A.$$

- To determine coordinates of $G(x_G; y_G)$ the center of triangle ABC use the following relations:

$$x_G = \frac{x_A + x_B + x_B}{3}.$$

$$y_G = \frac{y_A + y_B + y_C}{3}.$$

- Use properties of centroids to prove the above relations.

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