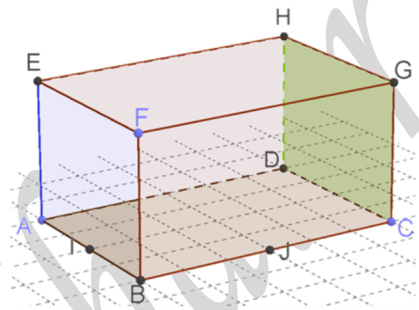
Reviewing Vectors

App-1:

The adjacent figure is a rectangular prism $ABCDEFGH$, where I & J are the respective midpoints of $[AB]$ & $[BC]$.



1) Observe the adjacent figure and complete the following table with the most convenient data.

Determine	Answers in vector form	Definitions and Conclusions
Zero vectors		
Collinear vectors		
Equal vectors		
Opposite vectors		
The sum: $\overrightarrow{AF}+\overrightarrow{DC}+\overrightarrow{FD}$		
The sum: $\overrightarrow{DE}+\overrightarrow{DC}$		
The sum: $\overrightarrow{AE}+\overrightarrow{BD}+\overrightarrow{HG}$		

2) Are the properties of vectors in space preserved? ……………………………………………..
3) If $\vec{u}$ & $\vec{v}$ are any two non-zero vectors and α is any real number, then what does the vector equality, $\vec{u}=\alpha\vec{v}$, tell you about the vectors $\vec{u}$ & $\vec{v}$, ***discuss***.

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4) Consider the reference frame $(A,\overrightarrow{AB},\overrightarrow{AD})$
   a. Find the coordinates of the points A, B & J and the vector $\overrightarrow{AC}$

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   b. Determine the equation of the straight line (d) defined by: $(B,\overrightarrow{AC})$ (***use vectors***)

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   c. Do vectors $\overrightarrow{AC}$ & $\overrightarrow{IJ}$ form a base? Justify. …………………………………………

Property: All properties of plane geometry are applied in space geometry.

Coplanar Vectors

Definition: Three vectors $\vec{u}, \vec{v} \,\&\, \vec{w}$ are coplanar ***iff***:

- One of the vectors can be written as a linear combination of the other two vectors: $\vec{w} = \alpha\vec{u} + \beta\vec{v}$, where $\alpha \,\&\, \beta$ are real.

App-2:

$ABCDEFGH$ is a cube of side $4cm$, where K is the center of $ABCD$, N is the midpoint of $[AB]$ and $\overrightarrow{AR} = \frac{1}{4}\overrightarrow{CG}$

1) Trace the figure and ***complete whenever needed***.
2) Prove that: $\overrightarrow{EG} = \overrightarrow{AB} + \overrightarrow{AD}$

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3) Are the vectors $\overrightarrow{EG}, \overrightarrow{AB} \,\&\, \overrightarrow{AD}$ coplanar? Justify.

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4) Are the vectors $\overrightarrow{AF}, \overrightarrow{AR} \,\&\, \overrightarrow{AN}$ coplanar? Justify

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5) Show that $\overrightarrow{DE}$ is parallel to the plane (BCG)……………………………………………………

   a. Prove that: $\overrightarrow{DE} = \alpha\overrightarrow{CG} + \beta\overrightarrow{BC}$. Where $\alpha \,\&\, \beta$ are two real numbers to be determined.

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   b. What can you say about the vectors $\overrightarrow{DE}, \overrightarrow{CG} \,\&\, \overrightarrow{BC}$? …………………………………………...

   c. Are the vectors $\overrightarrow{NK}, \overrightarrow{EH} \,\&\, \overrightarrow{EG}$ coplanar? Justify. ………………………………………….

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   d. What do you conclude? ...…………………………………………………………….................

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Def: Three vectors $\vec{u}, \vec{v} \,\&\, \vec{w}$ are coplanar ***iff***: Two of these vectors are collinear.

6) Can you find a way to prove that the four points $B, D, G \,\&\, K$ are coplanar

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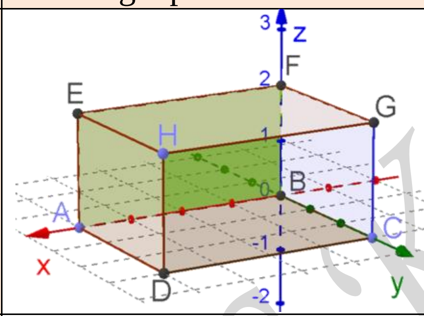
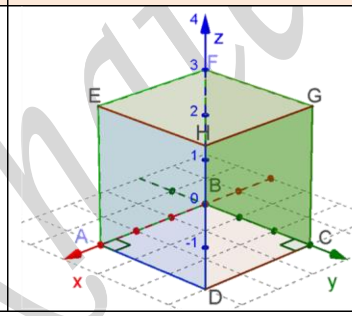
Base & system in space

Definition:	If $\vec{i}\,,\vec{j}\ \&\ \vec{k}$ are any three non-zero and non-coplanar vectors then, all triples $(\vec{i}\,,\vec{j},\vec{k})$ is called a base in space. Hence $(O,\vec{i}\,,\vec{j},\vec{k})$, is called a system in space, where O is the origin.

App-3:

Consider the	Rectangle prism $ABCDEFGH$	Cube $ABCDEFGH$
Questions		
Do the vectors $\overrightarrow{AB},\overrightarrow{AD}\ \&\ \overrightarrow{AE}$ form a system in space?		
Determine the type of the system $\left(B,\overrightarrow{BA},\overrightarrow{BC},\overrightarrow{BF}\right)$		
Determine the coordinates of: $A,D,G\ \&\ H$; $\overrightarrow{BD},\overrightarrow{FH}\ \&\ \overrightarrow{CE}$		
Write in analytic form: $\overrightarrow{BH},\overrightarrow{BG}\ \&\ \overrightarrow{BE}$		

App-4: Consider the tetrahedron $ABCD$ and the points $I,J,K\ \&\ L$ defined by:

$$\overrightarrow{AI}=\frac{1}{3}\overrightarrow{AB},\ \overrightarrow{CJ}=\frac{1}{3}\overrightarrow{CB},\ \overrightarrow{DK}=\frac{1}{4}\overrightarrow{DC}\ \&\ \overrightarrow{DL}=\frac{1}{5}\overrightarrow{DA}.$$

1) Draw figure and place the points.
2) Calculate the coordinates of the points $I,J,K\ \&\ L$ in the system $\left(A;\overrightarrow{AB},\overrightarrow{AC},\overrightarrow{AD}\right)$.

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3) Show that the points $I,J,K\ \&\ L$ are non-coplanar.

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Riminders

If $A(1;2;3)$ & $B(-2;3;1)$ are any two points and $\vec{v}(1,-1,3)$ is a vector in space then:

1) The coordinates of $\overrightarrow{AB}$ are: ……………………………………………………………
2) The norm of $\overrightarrow{AB}$ is: $\left\|\overrightarrow{AB}\right\| =$ ………………………………………………………………
3) Are $\overrightarrow{AB}$ & $\vec{v}$ collinear? ……………………………………………………………………..
4) Determine the angle formed between the straight lines (AB) & (l) of director vector $\vec{v}$.
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Equation of a st. line in space

App-5: Find the parametric equations of the straight (d) passing through the point $A(-1;2;3)$ and of directing vector $\vec{v}(2,-1,1)$.

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Equation of a plane

App-6:

Let A, B & C be any three non collinear points and $M(x, y)$ be any point.

1- Can the points A, B & C determine a plane? Justify. ………………………………………………

2- What is the geometric meaning of the following vector equations?

a) $\overrightarrow{AM} = k\overrightarrow{AB}$, where $k \in \mathbb{R}$. ……………………………………………………………………..

b) $\overrightarrow{AM} = \alpha\overrightarrow{AB} + \beta\overrightarrow{AC}$, where α & $\beta \in \mathbb{R}$. ……………………………………………………………

3- Determine the set of points M so that:

a. $\overrightarrow{AM} = k\overrightarrow{AB}$, where $k \in \mathbb{R}$. ……………………………………………………………………..

b. $\overrightarrow{AM} = \alpha\overrightarrow{AB} + \beta\overrightarrow{AC}$, where α & $\beta \in \mathbb{R}$. ……………………………………………………………

4- What do you need to find equation of a straight line?

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5- Is it true that a plane can be determined by a point and any two vectors?

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