Al Mahdi High Schools	*Mathematics*	*11^{th}-Grade*
Name:	*"Counting techniques"*	*A.S-14*

Focusing event:

1) 20 executives enter a room for a conference. Every person shakes hands with every other person. How many handshakes take place?
2) A committee of 4 is randomly chosen from 8 men and 7 women. What are the ***chances*** that two of each gender are chosen?

I- *Couples:*

Ex-1: Are the couples $(a;-3)$& $(-3;a)$ equal? ....

1- What is the type of the sets: $E=\{2,3\}$ *and* $F=\{3,2\}$. ....
2- Are the sets E& F equal? Justify. ....
3- What is the difference between a ***couple*** and a ***pair set***? ....

Def	A couple denoted by $(x;y)$, is an ....elements x & y, where x is the 1st element and y is .... the couple.

In set notation: $A\times B=\{(a;b)|\ a\in A\ \&\ b\in B\}$

Properties:

- The couple $(x;y)=(y;x)$ iff $x=y$
- If $(x;y)=(a;b)$, then $x=a$ & $y=b$ and vice versa.

II- *Cartesian product:*

Def	For all non-empty sets A & B. The set of couples $(x;y)$where $x\in A$ & $y\in B$ is called the cartesian product of the sets A & B.

Ex-2: Consider the sets $A=\{1,2,3\}$ & $B=\{blue, green\}$.

Find the following Cartesian products:

1) $A\times B=$	$B\times B=$	$Card(A\times B)=$
2) $B\times A=$	$Card(A)=$	$Card(B\times A)=$
3) $A\times A=$	$Card(B)=$	$Card(A\times A)=$

❖ *Properties of cartesian product:*

1- For all $set\ A\neq set\ B$, then $A\times B$ ... ... $B\times A$
2- $Card(A\times B)=Card(B\times A)=$ ....
3- If $A\times A$ is denoted by: A^2, then $A\times A\times A$ is denoted by: ....

Ex-3: A basketball coach is trying Ali, Nader, Sami and Jamal for one of the following positions, forward, center and guard.

a) Write in extension the set of: - New players, $T=\{\qquad\qquad\}$
- Expected positions, $P=\{\qquad\qquad\}$
b) List all the possible choices for the coach as a couple $(x;y)$, ***Where***: x is the name of the player and y is the expected position. ....

....

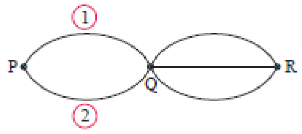
c) Construct a tree diagram to show all possible choices (outcomes) of the coach:

A	N	S	J

d) Count the number of choices. ……………………………………………
e) Can you device a way to find the number of possible choices without listing them? Explain.………………………………………………………………………………

III- Product principle :

There are three towns, $P, Q \& R$, that are linked by roads as shown.
How many different ways are there to get from $P\ to\ R\ via Q$?



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How many different ways are there to get from $P\ to\ S\ via Q \& R$?

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If each boy gives a gift to every other boy in the park, then find the number of gifts exchanged among 24 boys.
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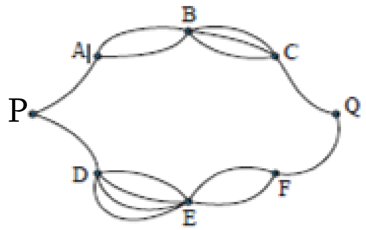
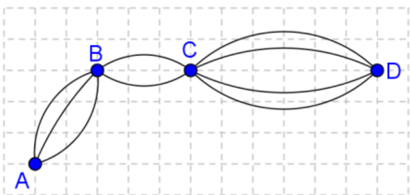
Def	If there are m different ways to perform an operation, and for each of these there are n different ways of performing a second independent operation, then there are ……………………………………………………. in succession.

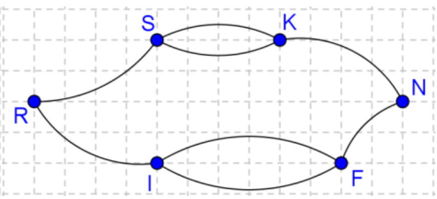
IV- The sum principle:
Consider the road system from $P\ to\ Q$.



1) How many path ways are there from:
   a. $A\ to\ C\ via B$? ………………………………………………..
   b. $D\ to\ F\ via E$? ………………………………………………..
2) Deduce in how many ways can you pass from $P\ to\ Q$. ……………………………………
3) What does each of the following words suggest to the available possibilities?
   a. And: ………………………………………………………………………………
   b. Or: ………………………………………………………………………………

Consider the following pathways:





Describe in words, the difference between the above pathways.(Don't find the numerical value)

………

………

V- Counting techniques:

a) Counting with permutation:

Def	A permutation of a group of n objects: is any arrangement of those n objects in a ***definite order***.

b) Counting with combinations:

Def	A combination is a selection of objects without regard to the order or arrangement.

How many teams of three can be selected from the set $P =$.

c) *Arrangement:* we distinguish two cases

i. ($\mathcal{P}$ - *list*): is an arrangement ***with repetition*** of a group of elements.

Ex-1: Consider the set $E = \{a, b, c\}$

a. Write all possible arrangements of ***two elements*** of set E. Repetition is allowed.

- In a couple form: ..……………………………………………………………………

- In a tree form:

b. Deduce the number of formed arrangements. ………………………………………

Ex-2: ***<u>The following questions are independent:</u>***

1) Find the number of all:
   a. 3-list out of 2 choices. ……………………
   b. 5-list out of 3 choices. ……………………
2) What is the number of all expected outcomes, for throwing:
   a. Three fair coins? ………………………
   b. Two fair dices? ………………………
   c. How many six digit secret combinations can you form, if all digits are numbers? ………………………………………………………………………………...............
   d. How many 4-digit numbers can be formed, using the elements of the set: $D = \{0, 1, 2, 3, 4\}$? ………………………………………………………

ii. Arrangement without repetition:

❖ Summary of counting techniques:

Counting technique	Arrangement		Permutation	Combination
	With repetition (p-list)	Without repetition	Without repetition	
Conditions	✓ Order is important. ✓ Action is taken in succession.			✓ Order is not important. ✓ Simultaneous action
Number of possibilities (outcomes)	n^p	$A_n^p = \dfrac{n!}{(n-p)!}$	$P_n = A_n^n = n!$	$C_n^p = \dfrac{A_n^r}{p_r} = \dfrac{n!}{r!(n-r)!}$

Probability:

In a group of 18 students, eight are females. What is the probability of choosing five students where:

1. All girls?
2. Three girls and two boys are selected?
3. At least one boy is selected?

Solution:

The total number of outcomes, is the number of ways we can choose 5 out of the 18 students.
So, $Card(\Omega) = A_{18}^{5}$

a) Let A be the event of "picking up girls only".
   $Card(A) = A_{8}^{5}$
   Thus, $P(A) = \dfrac{Card(A)}{card(\Omega)}$
b) Let B be the event of "selecting 3 girls & 2 boys"
   $Card(B) = A_{8}^{3} \times A_{10}^{2}$
   Thus, $P(B) = \dfrac{Card(B)}{card(\Omega)}$
c) Let C be the event of choosing " at least one boy".
   1st method: $Card(C) = A_{10}^{1} \times A_{8}^{4} + A_{10}^{2} \times A_{8}^{3} + A_{10}^{3} \times A_{8}^{2} + A_{10}^{4} \times A_{8}^{1} + A_{10}^{5} \times A_{8}^{0}$
   2nd method: Since $A$is the complement event of C
   so, $P(A) + P(C) = 1$