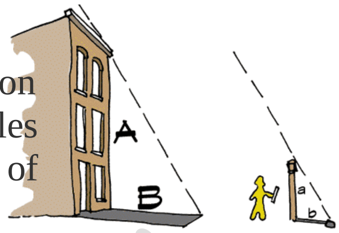
Introduction:



The building of the Egyptian pyramids may seem to have little in common with modern sciences like ***geophysics*** & ***seismology***. But certain principles of mathematics enter into all such activities. Many are used in the field of mathematics called ***Trigonometry***.

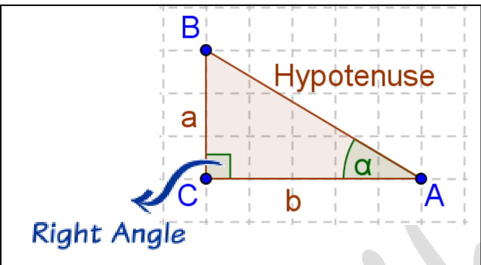
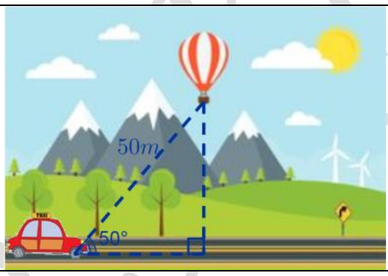
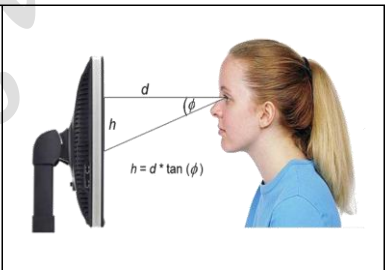
A. *What does Trigonometry do?*

Trigonometry finds ***relationships*** between the ***sides*** and ***angles*** of a right triangle.

B. *Goals:*

1- *Find length of a side, establish identities*
2- *Determine the value of an acute angle between any two segments.*
3- *Find the area of a triangle using an angle enclosed between two given sides.*

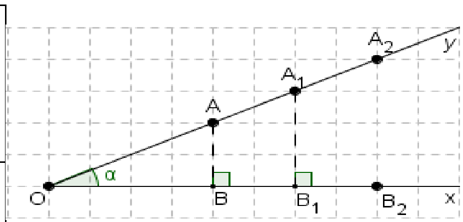
Focusing events:

Figures			
Questions	Label the sides of the above right triangle with respect to α	Find the exact height of the balloon from ground.	Do you know why it is better to step away from a TV screen?
Possible answers	a: .... b:		

C. *Defining the three main trigonometric ratios:* $\sin\alpha; \cos\alpha;$ *and* $\tan\alpha$.

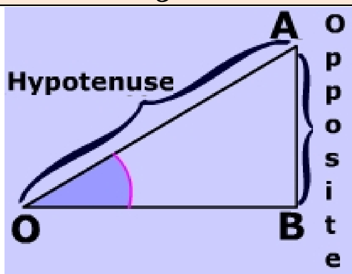
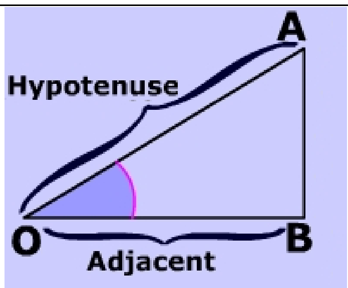
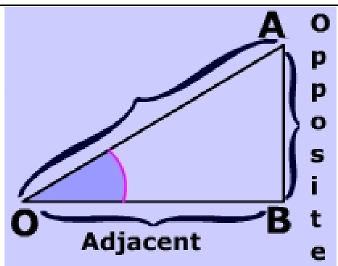
Let B be the orthogonal projection of a variable point A, that belongs to ray $[oy)$, on $[ox)$. Where $x\hat{o}y = \alpha$, if $[AB]$ varies in a way that it remains perpendicular to $[ox)$, then prove that

1) $\frac{OB}{OA} = \frac{OB_1}{OA_1} = \frac{OB_2}{OA_2} = cst$	
2) $\frac{AB}{OA} = \frac{AB_1}{OA_1} = \frac{AB_2}{OA_2} = cst$	



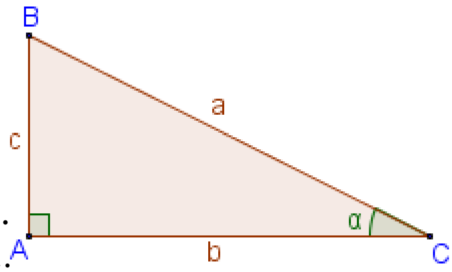
- ✓ If points $A \& B$ move again, will the ratios remain the same? ....
- ✓ What do sides $OA \& OB$ represent for ΔOAB? ....
- ✓ When will these ratios alter? ....

Conclusions:	If α is constant then, the ratio of: ❖ Adjacent to hypotenuse .... ❖ ……….. to hypotenuse remains ……………

Trigonometric line	*Figure*	*Formula*	*How to remember*
Sine		$\sin A\hat{O}B = \frac{\text{Opposite}}{\text{Hypotenuse}} = \frac{AB}{OA}$ **IN SHORT** $\sin\alpha = \frac{Opp}{hyp}$	Soh
Cosine		$\cos A\hat{O}B = \frac{Adjacent}{\text{Hypotenuse}} = \frac{OB}{OA}$ **IN SHORT** $\cos\alpha = \frac{adj}{hyp}$	Cah
Tangent		$\tan A\hat{B}C = \frac{\text{Opposite}}{Adjacent} = \frac{AB}{OB}$ **IN SHORT** $\tan\alpha = \frac{Opp}{Adj}$ or $\tan\alpha = \frac{\sin\alpha}{\cos\alpha}$	Toa *OR* tsc

D. *Fundamental trigonometric identities relating:*

1) Sine and Cosine: The Pythagorean identity:



a- Find $\sin\alpha$ & $\cos\alpha$

$\sin\alpha =$.......... $\cos\alpha =$..................

b- Write the Pythagoras theorem in the right triangle ABC.

……………………………………………………………………

c- Use parts a & b to verify that: $\sin^2\alpha + \cos^2\alpha = 1$

……………………………………………………………………

……………………………………………………………………

……………………………………………………………………

……………………………………………………………………

……………………………………………………………………

Therefore, $$\sin^2\alpha + \cos^2\alpha = 1$$

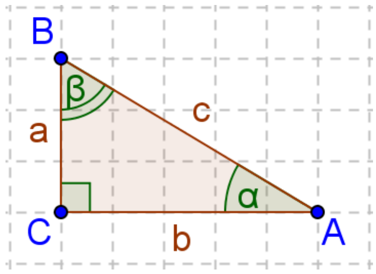
2) Cosine and tangent: Use the Pythagorean identity, $\sin^2\alpha + \cos^2\alpha = 1$ to find:

a) $\tan\alpha$ in terms of $\cos\alpha$: ………………………………………………

b) $\cos\alpha$ as a function of $\tan\alpha$: ………………………………………………

Therefore, $$\tan^2\alpha = \frac{1-\cos^2\alpha}{\cos^2\alpha}$$ **and** $$\cos^2\alpha = \frac{1}{1+\tan^2\alpha}$$

Consider the right triangle ABC of hypotenuse $[AC]$



1) Indicate the sum of α & β : ……………………………………

2) What do we call α & β : ……………………………………

3) Determine in terms of sides:

$\sin\alpha =$	$\cos\beta =$
$\sin\beta =$	$\cos\alpha =$

4) Compare obtained results: ……………………………………..

Conclusions: If α & β are complementary angles, then ……………………………………

Complementary Angles

E. *Remarkable angles:*

a) Use your calculator to complete:

Angle (α) / Ratios	0°	30°	45°	60°	90°
$\sin\alpha$			$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}=1$
$\cos\alpha$				$\frac{\sqrt{1}}{2}=\frac{1}{2}$	$\frac{\sqrt{0}}{2}=0$
$\tan\alpha$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	∞
$\cot\alpha$					

b) Can you device a method to memorize the above values in ease? Try.

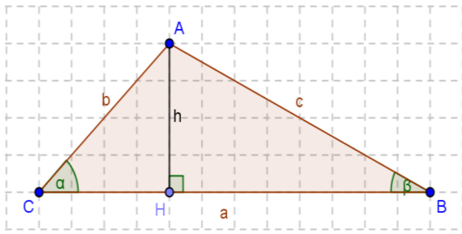
c) Use the above table to complete if α & β are two ……………… angles (α + β=90°) then,

$\cot\alpha \times \cot\beta = ...$

$$\tan\alpha = \cot\beta$$

$\tan\alpha \times .... = 1$

F. *Area of a triangle in a different way .(Using sine of the angle enclosed between two sides):*



1) Indicate the area of the triangle ABC . ………………

2) Find $\sin\alpha$. ……………………………………………

3) Determine the area of ABC independent of h .

……………………………………………………………

4) Can you find the area of ABC as a function of a, c & β .

……………………………………………………………

5) Compare obtained results: ……………………………………..

Conclusions: The area of a triangle is equal to one-half the product of two sides and the sine of the contained angle.

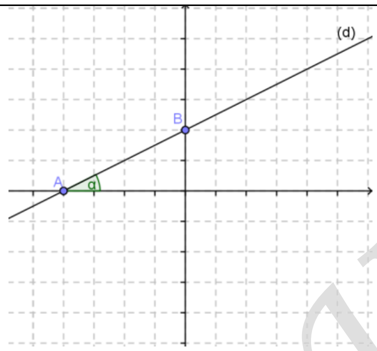
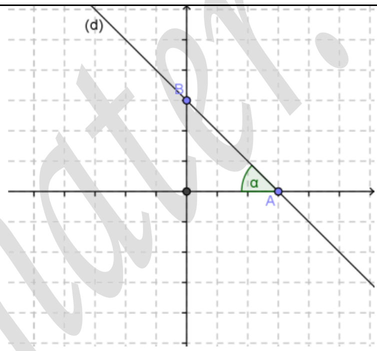
In symbols:

$$Area = \frac{ab\sin\alpha}{2} = \frac{ac\sin\beta}{2}$$

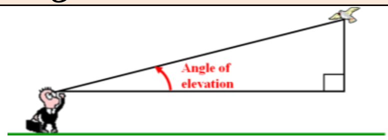
G. Bounding (Framing) trigonometric ratios:

In words	***sine*** and ***cosine*** ratios of an ***acute angle*** are bounded between 0 and 1		***tangent*** and ***cotangent*** ratios of an ***acute angle*** are greater than zero	
In symbols	$0 \leq \sin\alpha \leq 1$	$0 \leq \cos\alpha \leq 1$	$\tan\alpha \geq 0$	$\cot\alpha \geq 0$

H. Trigonometry and coordinate system:

Sense of the straight line	***Increasing***	***Decreasing***
Graphical representation		
Form of the slope	$slope = \tan\alpha$	$slope = -\tan\alpha$
Condition on the angle α	where α is the acute angle between the given line and the ***positive*** *x*-axis	α is the acute angle between the given line and ***negative*** the *x*-axis

I. Real life situations:

Angle of	*Definition*	*Figure*
Elevation	Is the acute angle formed ***above*** the horizontal to inspect an object.	
Depression	Is the acute angle formed ***below*** the horizontal to inspect an object.