Goals: 1) Sign of the 2nd degree polynomial.
2) Sign of the roots.

1) Studying the sign of the quadratic trinomial: $f(x) = ax^2 + bx + c$.

I- Consider the following quadratic expressions:

1) $A(x) = -1(x-3)^2$
2) $B(x) = 1(x+2)^2$

A- Analytical study if $\Delta = 0$:

a) Find roots for the given expressions:

……..

……..

b) Deduce without calculation, for the above expressions:

i. Discriminant:

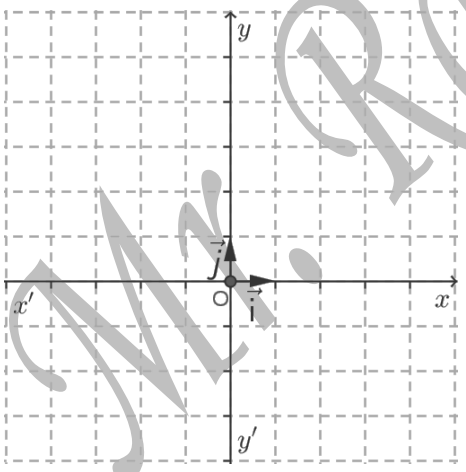
- ……..
- ……..

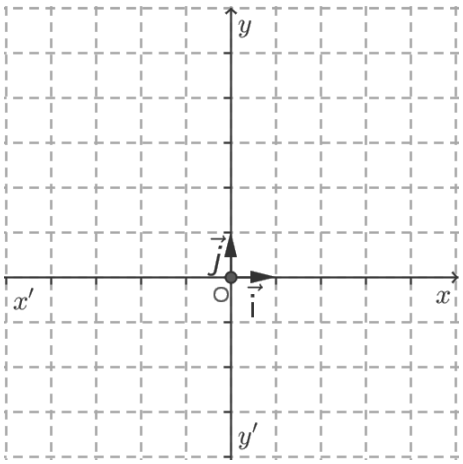
ii. The sign. Justify:

- ……..
- ……..

B- Graphical study if $\Delta = 0$:

c) Graph $A(x)$ & $B(x)$





d) What is the sign of $A(x)$ & $B(x)$ at their roots? ……

……

Conclusion: If $f(x)$ is a 2nd degree trinomial of the form: $f(x) = \alpha(ax-b)^2$, where $\alpha \neq 0$

then the sign of $f(x)$ ……..

ℂ- Analytical study if $\Delta<0 \& \Delta>0$:

Consider the functions $f, g \& h$ defined by:

- $h(x)=x^2-5x+4$ $\qquad f(x)=x^2-2x+5$
- $g: x \longrightarrow -x^2-4x+5$ $\qquad k(x)=-x^2+2x-2$

1- Express each of the above trinomials in product form if possible.

…….. ……

…….. ……

…….. ……

…….. ……

2- Study the sign of each of the above trinomials.

h: $\qquad$ f:

g: $\qquad$ k:

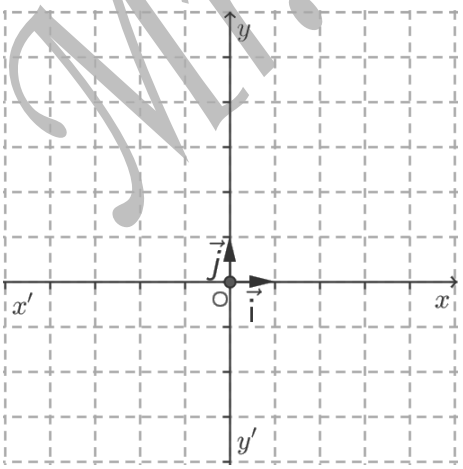
3- Indicate for each of the above trinomials the sign of :

	The leading coefficient	The discriminant
$h(x)$		
$g(x)$		
$f(x)$		
$k(x)$		

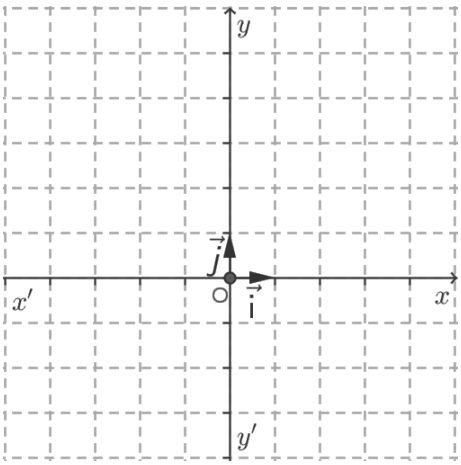
4- What do you conclude? ……..

……

5- On the orthonormal system of axes $\left(O, \vec{i}, \vec{j}\right)$ trace the graphs of $C_f, C_g, C_h \& C_k$.



$\Delta>0$



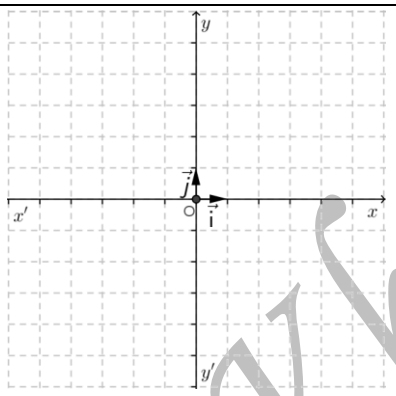
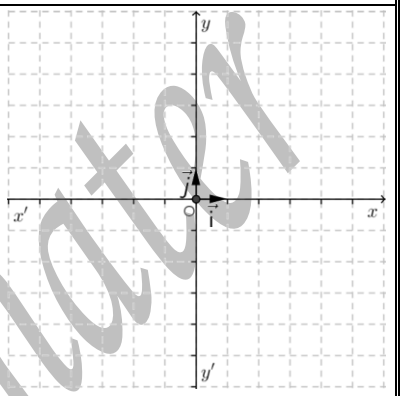
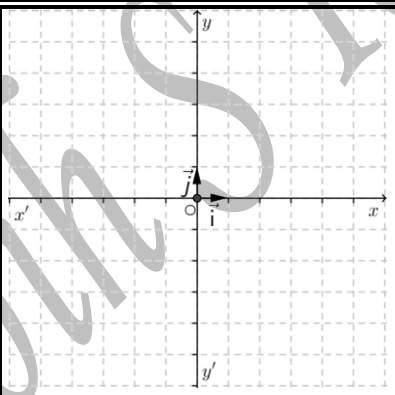
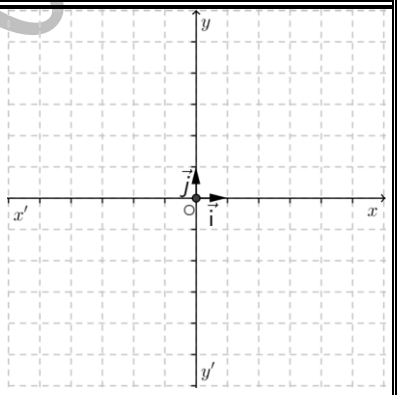
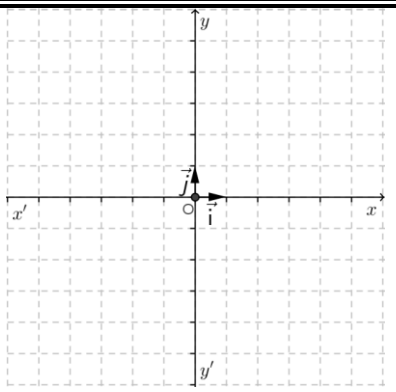
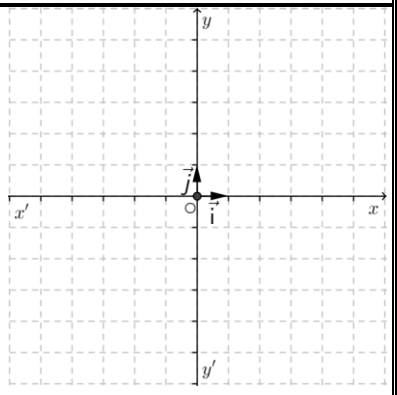
$\Delta<0$

6- What do you conclude?

Summary

Let x_1 & x_2 be the roots of the second trinomial $f(x) = ax^2 + bx + c$ if they exist.

Complete the following table:

If		*Then*	
Δ	*Sign of a*	*Sign of the trinomial* $f(x) = ax^2 + bx + c$	*Graphically*
$\Delta = 0$	$a < 0$		 $a < 0$
	$a > 0$		 $a > 0$
$\Delta > 0$	$a < 0$		 $a < 0$
	$a > 0$		 $a > 0$
$\Delta < 0$	$a < 0$		 $a < 0$
	$a > 0$		 $a > 0$

- On what factors does the sign of a quadratic expression depend?
  - ✓ ……..
  - ✓ ……..

2) Sign of the roots of a 2nd degree trinomial:

Let x_1 & x_2 be the roots of the 2nd degree trinomial: $f(x) = ax^2 + bx + c$. where S & P represent the sum and the product of the roots of $f(x)$ respectively.

a) When would $f(x)$ admit two real roots? ……...

b) Write S & P in terms of the:

   a. Roots: ……...

   b. Coefficients: ……

c) Consider the following different representations of $f(x)$:

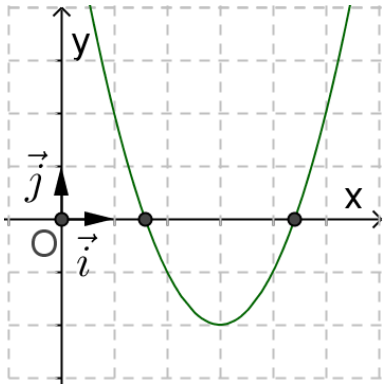


Fig-1.

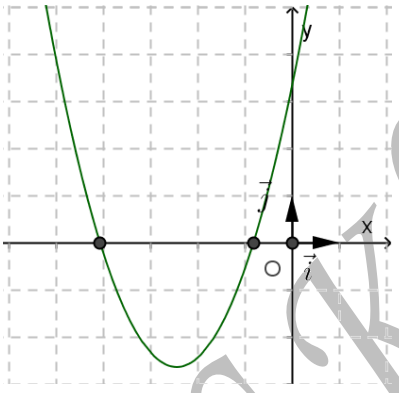


Fig-2.

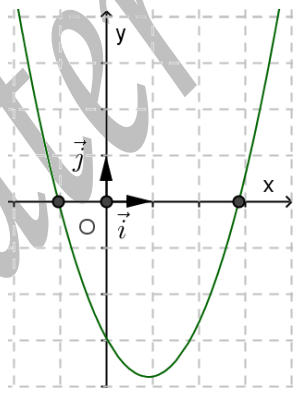


Fig-3.

d) Determine the sign of the roots in each of the above figures:

   i. Fig-1: ……...

   ii. Fig-2: ……...

   iii. Fig-3: ……...

e) Determine the sign of S & P in each of the above figures:

   i. Fig-1: ……...

   ii. Fig-2: ……...

   iii. Fig-3: ……...

f) Deduce when would $f(x)$ admit two opposite roots. ……

💣 *Conclusion:* A second degree trinomial in one unknown admits:

Statement		*Conditions*
Two positive roots: $0 < x_1 \le x_2$	*If*	
		$S < 0$ & $P > 0$
Two roots of opposite signs: $x_1 < 0 < x_2$		
Two opposite roots		