Lycée Des Arts

Name: . . . .

Mathematics

"The Four Tangent Theorems"

*Tangents to a circle through an exterior point*

9th-*Grade*

A.S-4.

Let (C) be a circle of center O and radius $r\ cm$, where A is any point in the plane.

1- How many tangents can be drawn from a point A:

     a) Inside the circle? ………………………   c) Outside the circle? ………………………..
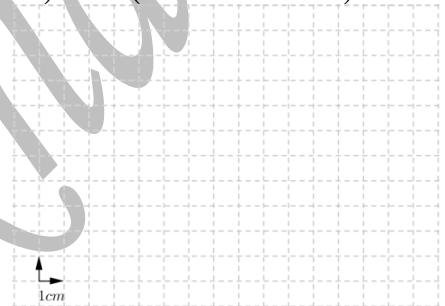
     b) On the circle? ……………………………

2- Let (T) be the tangent at a point A on the circle (C) of center O.

     a. What is the value of the angle formed by the straight line (T) and the radius $[OA]$? ……………………………..

     b. Construct (T)

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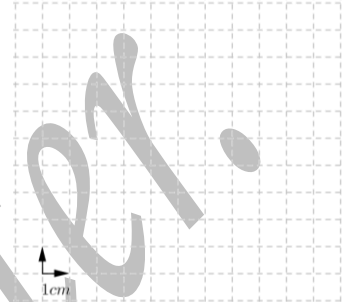
3- Let $A\ \&\ B$ be the points of intersection of the circles $s(O,3cm)\ \&\ n(O',OO'=4cm)$.

     a. Trace the circles and plot the points $A\ \&\ B$.

     b. Plot P the symmetric of O with respect to O'.

     c. What do the angles $P\hat{A}O\ \&\ P\hat{B}O$ represent?

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     d. Deduce the measure of $P\hat{A}O\ \&\ P\hat{B}O$.

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4- Consider the circle $\psi(O,3cm)$ and the point A so that $OA = 5cm$

     a. Devise a method to trace the tangents $(T_1)\ \&\ (T_2)$ from A to (ψ) at $R\ \&\ N$ respectively.

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     b. What does each of the following represent:

       i. $R\hat{A}N$: …………………………………............

       ii. $R\hat{O}N$: …………………………………............

       iii. $[RN]$: …………………………………............

     c. Prove that: $[AO)$ is the bisector of $R\hat{A}N$: ……………………………………………

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     d. Deduce that $[OA)$ is the bisector of $R\hat{O}N$: ……………………………………………..

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     e. What is the relative position of (OA) with respect to $[RN]$?

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Conclusions

If (*AR*) & (*AN*) are two tangents drawn from a point *A* to a circle (*C*) of center *O* at *R* & *N* resp.
Then,

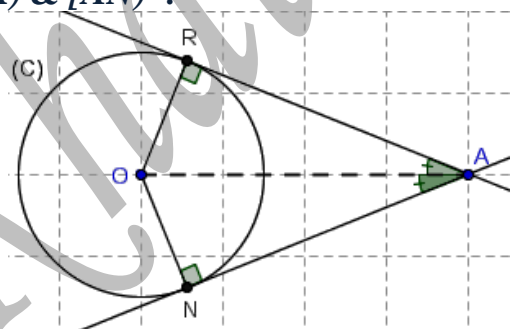**1. A is equidistant from the two points of tangencies R & N**

> Or, $AR = AN$.
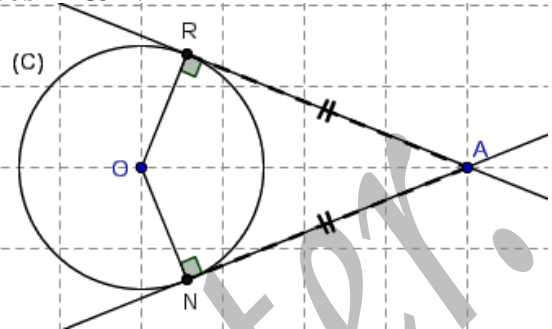
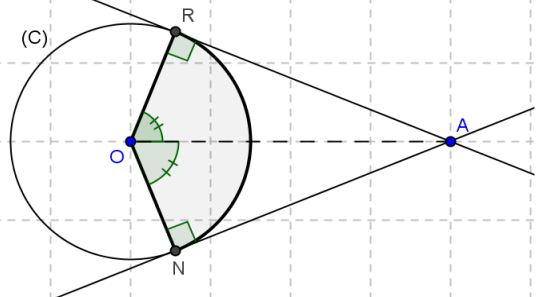**2. Line joining point of intersection of the two tangents and center of (C) bisects:**

    a. The angle formed by the two tangents "[AR) & [AN)".

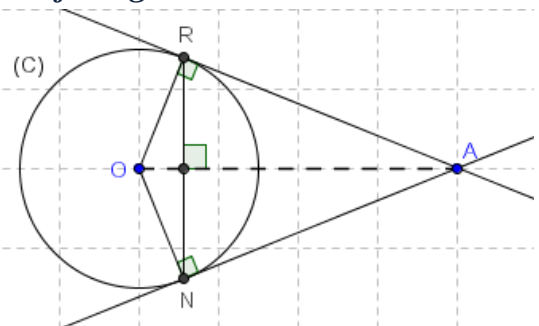> Or, $[AO)$ is the bisector of the angle $R\hat{A}N$.

    b. The central angle intercepting the arc formed by the two points of tangencies.

> Or, $[OA)$ is the bisector of the angle $R\hat{O}N$.

3. Line joining point of intersection of the two tangents and center of (C) is the perpendicular bisector of the chord joining points of tangencies.

> Or, (OA) is the $\perp$ bisector of $[RN]$

4. **Line joining point of intersection of the two tangents and center of (C) is the axis of symmetry of the figure formed by (C) and the two tangents.**