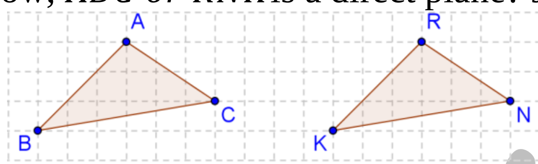
Al Mahdi High Schools	Mathematics	11^{th}-Grade
Name:	" Oriented Angles "	A.S-5.

A. Definitions and over view:

- ✓ A plane is oriented if every circle in it is oriented.
- ✓ A triangle ABC is direct if and only if the principal measure of $(\overrightarrow{AB}\,;\,\overrightarrow{AC})$, is stirctly positive.

a) Which of the triangles below, ABC or RNK is a direct plane? Justify.

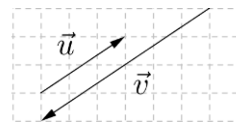


…………………………………………………………………………………………………...

b) Find the following sum: $(\overrightarrow{AB}\,;\,\overrightarrow{AC}) + (\overrightarrow{BC}\,;\,\overrightarrow{BA}) + (\overrightarrow{CA}\,;\,\overrightarrow{CB})$= ………………………………

- ✓ Unit vectors: $\vec{v}$ is a unit vector if and only if its norm is 1 unit. And we write: $\|\vec{v}\| = 1\ unit$.

1) Consider the vectors $\vec{u}$ & $\vec{v}$:



   a. What can you say about the $\vec{u}$ & $\vec{v}$? ……………………………………….
   b. Write the vector equation that relates the above vector. ………………..

2) Let $\vec{u}$ & $\vec{v}$ be any two non-zero vectors, where $\vec{u} = k\vec{v}$ and $k \in \mathbb{R}^*$

Discuss according to the values of k the orientation of $\vec{u}$ & $\vec{v}$:

If	Then
$k = 1$	$\vec{u} = \vec{v}$

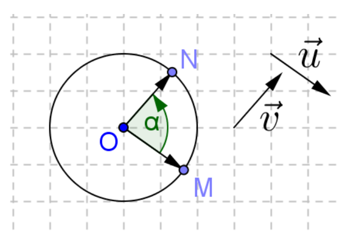
3) Consider in the orthonormal system of axes $\left(O;\vec{i},\vec{j}\right)$ the points $A(3;2)$ & $B(5;1)$:

a. Determine: $i)$ $\overrightarrow{AB}$: ……………………………………

$ii)$ $\left\|\overrightarrow{AB}\right\|$: ……………………………………

b. Determine two unit vectors that are collinear with $\overrightarrow{AB}$: ……………………………………

B. How to find the angle between any two free vectors:

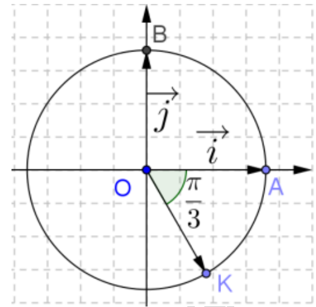


Let $\vec{u}$ & $\vec{v}$ be any two non-zero vectors so that, $\vec{u} = \overrightarrow{OM}$ and $\vec{v} = \overrightarrow{ON}$:

If $M\hat{O}N = \alpha + 2k\pi$, where $k \in \mathbb{Z}$, then the angle formed between $\left(\vec{u},\vec{v}\right) =$ ………

Or we can write, $M\hat{O}N = \alpha \bmod [2\pi]$.

Ex: In the orthonormal system $\left(O;\vec{i},\vec{j}\right)$, plot the points:



- ✓ E the midpoint of $\widehat{AB}$.
- ✓ G so that $\left(\overrightarrow{OB},\overrightarrow{OG}\right)=\frac{\pi}{6}\bmod[2\pi]$.
- ✓ B' which is diametrically opposite to B.

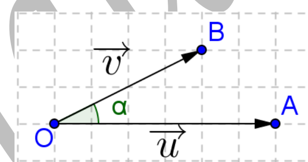
Determine the angles:

$\left(\overrightarrow{OA},\overrightarrow{OE}\right)=$............................ $\left(\overrightarrow{OE},\overrightarrow{OG}\right)=$............................

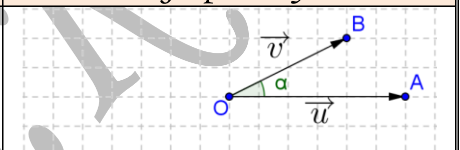
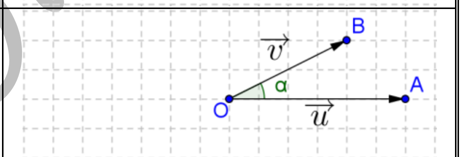
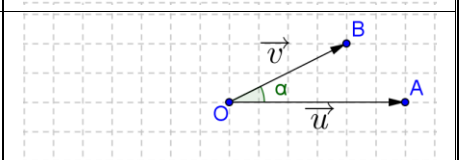
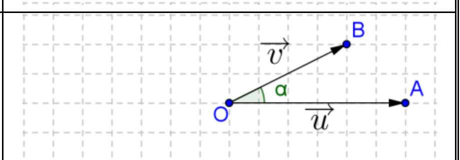
$\left(\overrightarrow{OE},\overrightarrow{OB'}\right)=$............................ $\left(\overrightarrow{OE},\overrightarrow{OK}\right)=$............................

C. Angle between two non-zero vectors:

Let $\vec{u}$ & $\vec{v}$ be any two non-zero vectors so that, $\left(\vec{u},\vec{v}\right)=\alpha+2k\pi$



Determine the following angles:

Analytically	*Graphically*
$\left(\vec{u},-\vec{v}\right)=$	
$\left(-\vec{u},\vec{v}\right)=$	
$\left(-\vec{u},-\vec{v}\right)=$	
$\left(\vec{v},\vec{u}\right)=$	

Properties:

- ✓ $\left(\vec{u},\vec{v}\right)=\left(a\vec{u},a\vec{v}\right)$ for all $a\neq 0$.
- ✓ $\left(\vec{u},\vec{v}\right)=\left(a\vec{u},b\vec{v}\right)$ for all $ab>0$
- ✓ $\vec{u}$ *and* $\vec{v}$ are collinear if and only if $\left(\vec{u},\vec{v}\right)=k\pi$, $\begin{cases} \text{if } k<0, \text{then vectors are of opp. senses.} \\ \text{if } k>0, \text{then vectors are of same senses.} \end{cases}$

Ex: Determine $\left(\overrightarrow{CA};\overrightarrow{CB}\right)$, if ABC is a direct triangle such that : $\left(\overrightarrow{AB};\overrightarrow{AC}\right)=\frac{5\pi}{12}$ and $\left(\overrightarrow{BA};\overrightarrow{BC}\right)=-\frac{\pi}{6}$.

Chasle's relation:

- ❖ $\left(\vec{u},\vec{v}\right)+\left(\vec{v},\vec{w}\right)=\left(\vec{u},\vec{w}\right)$