Focusing event:

- To determine the size of a TV screen one has to measure the diagonal of the screen. If the size of a screen is32 *inch* it means that the diagonal of the screen is 82.28 *cm*. I wonder how many centimeters is a 50 *inch* screen?

  ……………………………………………………………………………………………..

Introduction:

Dealing with proportional numbers is just like equivalent fractions

Ex:$\frac{3}{5}=\frac{3\times2}{5\times2}=\frac{3\times3}{5\times3}=\frac{3\times4}{5\times4}=\cdots$ give two or fractions that are equivalent to $\frac{3}{4}=\cdots=\cdots=\cdots$

Def₁: A sequence $(y_1, y_2, y_3 ...)$ of non-zero real numbers is ***directly proportional*** to another non-zero set of numbers $(x_1, x_2, x_3 ...)$ if and only if: $\frac{y_1}{x_1}=\frac{y_2}{x_2}=\frac{y_3}{x_3}=\cdots=k$, where k is called the common ratio or ratio of of proportionality

Def₂: The magnitudes x & y of two sets of real numbers are directly proportional if and only if, the ratio of $\frac{y}{x}=a$ or we write: $y=ax$
Where, the relation $y=ax$ is called a ***linear function***

Use above definitions to answer the following questions

Ex₁: Consider the following proportionality tables:

Part-A:

Table-1					
	x	2	5	7	11
	y	6	15	21	33

1) Find the following ratios: $\frac{y_1}{x_1}=$……………; $\frac{y_2}{x_2}=$……………; $\frac{y_3}{x_3}=$……………; $\frac{y_4}{x_4}=$……………
2) What do you notice? ……………………………………………………………………..
3) Are x & y proportional? Justify. …………………………………………………………..
4) What constant, a, multiplied by x gives y? ………………………………………………
5) Write an algebraic expression of y as a function of x: ……………………………………..

Part-B:

Table-2				
	x	5	10	35
	y	1	2	7

i- Find the following ratios: $\frac{y_1}{x_1}=$……………; $\frac{y_2}{x_2}=$……………; $\frac{y_3}{x_3}=$……………

ii- What do you notice? ……………………………………………………………………

iii- What constant, a, multiplied by x gives y? ……………………………………………

iv- Are x & y proportional? Justify. ………………………………………………………..

v- Write an algebraic expression of y as a function of x: ……………………………………..

Ex2: Consider the following table:

x	12	33	35
y	4	11	7

a) Is the above table a proportionality table? Justify. ……………………………………………
b) Propose a definition for proportionality table: …………………………………………..
……………………………………………………………………………………………..

Ex3: Find the numerical values of $x \& y$, so that the table below is a proportionality table:

x	$\sqrt{\sqrt{5}+2}$	3	5
$\sqrt{\sqrt{5}-2}$	4	y	7

……………………………………………… ……………………………………………......
……………………………………………… ……………………………………………......
……………………………………………… ……………………………………………......

Proportionality and ratios:

If $a,b,c \& d$ are proportional numbers, then we can write in this order $\frac{a}{b}=\frac{c}{d}$

extreme → $\frac{a}{b}=\frac{c}{d}$ *← extreme*   *mean →* $\frac{a}{b}=\frac{c}{d}$ *← mean*

1) If the ***first*** and ***fourth*** terms ($a \& d$) are the ***extremes*** of the formed proportion, also the ***second*** and ***third*** terms ($b \& c$) are the ***means*** of the formed proportion, then find:
   - ✓ The ***product*** of the ***means:*** ……………………………………………
   - ✓ The ***product*** of the ***extremes:*** ……………………………………………
     - ➢ Are the formed products ***equal***? ………………………………
   - ✓ If we permute (interchange) the ***extreme*** terms, we obtain: $\frac{\square}{b}=\frac{c}{\square}$
   - ✓ Permute (interchange) the ***mean*** terms:
   - ✓ Determine the ***inverse*** of: $\frac{a}{b}=\frac{c}{d}$: ………………………..

> ☪ ***Fourth Proportional:*** x is the fourth proportional of the numbers a; b and c when $\frac{a}{b}=\frac{c}{x}$.
>
> ☪ ***Mean of Proportionality:*** x is the mean proportional of the numbers a and d when $\frac{a}{x}=\frac{x}{d}$.

Consider the following table:

x	1	4	$0.\bar{3}$
y	2	0.5	6

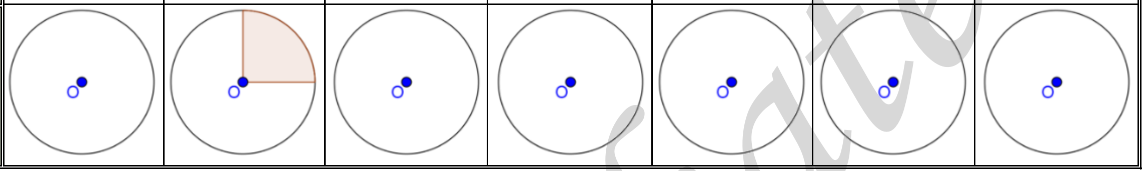
1) Is the above table a proportionality table? Justify.
………………………………………………………………………………………
2) Find the following products: $x_1 \cdot y_1=$……………; $x_2 \cdot y_2=$……………; $x_3 \cdot y_3=$……………;
3) What do you notice? ……………………………………………………………………
4) Write an algebraic expression that relates $x \& y$: ……………………………………………

Def2: A sequence $(y_1, y_2, y_3 ...)$ of non-zero real numbers is ***inversely proportional*** to another non-zero set of numbers $(x_1, x_2, x_3 ...)$ if and only if: $\frac{y_1}{\frac{1}{x_1}}=\frac{y_2}{\frac{1}{x_2}}=\frac{y_3}{\frac{1}{x_3}}=...=k$.

, ***Or we can write*** $x_1 y_1 = x_2 y_2 = \cdots = k$

Linear functions and Percentages:

Complete the following table:

Different representations of numbers (fractions)							
Fraction form	$\frac{1}{2}$					$\frac{3}{5}$	
Decimal form				0.2			
Percentage			75%				
In words	Half						One
Angle					135^o		360^o
Graphically							

Percentage of an item:

If r and n are any two positive numbers, then r % of n is given by the relation: $\frac{r}{100} \times n$

Increase in price:

If the initial price (size, measure …) x of an item is increased by r %, then:

1	The ***increase*** in price is given by: $a_1 = \left(1 + \frac{r}{100}\right)$
2	The ***Value*** of the new price ***y*** of the item will be given by: $y = x + \frac{r}{100}x$ ***OR*** $y = ax = \left(1 + \frac{r}{100}\right)x$

Decrease in price:

If the original price (length, area …) x of an item is decreased by a%, then:

1'	The ***decrease*** in price is given by: $a_2 = \left(1 - \frac{d}{100}\right)$
2'	The ***Value*** of the new price ***y*** of the item will be given by: $y = x - \frac{d}{100}x$ ***OR*** $y = ax = \left(1 - \frac{d}{100}\right)x$

Double decrease or increase in price: $y = (a_1.a_2)x$

Notes:

1) If the values x & y are directly proportional then as x ***increases*** y ***increases*** too.
2) If $a > 1$ where x & y belong to $\Re^+$ then, $y > x$ (New price is greater than old one)
3) If $a < 1$ where x & y belong to $\Re^+$ then, $y < x$ (New price is less than old one)
4) If $a = 1$ where x & y belong to $\Re^+$ then, $y = x$.

Graphical representation of a linear function:

Terminologies:

- f :is the rule that relates the variables x & y.
- x :is the pre-image of the given rule.
- y *or* $f(x)$: is the image of x by f.

f	x	0.5	1	2	3
	$y = f(x)$	1	2	4	6
	(x, y)	(0.5,1)	(1,2)	(2,4)	(3,6)

g	x	1	2	3
	$y = g(x)$	1.5	3	4.5
	(x, y)			

Consider the following tables of proportionalities:

1) Watch table-1, and complete the adjacent table-2.
2) Form the linear function that represent table-2:
   - Table-1: $f: y = 0.5x$
   - Table-2: ……………………………….
3) What does the ordered pair (x, y) represent graphically? (Point on the graph of the function)
4) Plot on the following orthonormal system of axes the points in table-2:

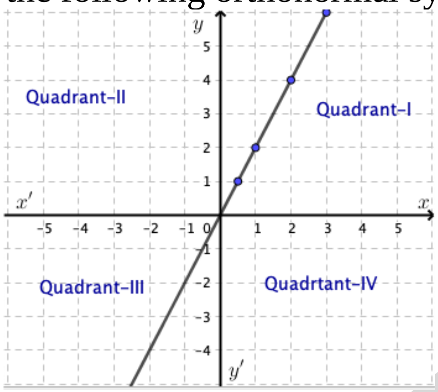


Table-1.

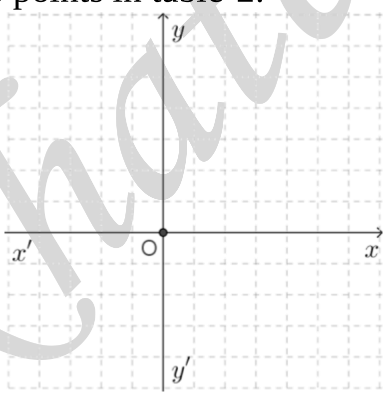


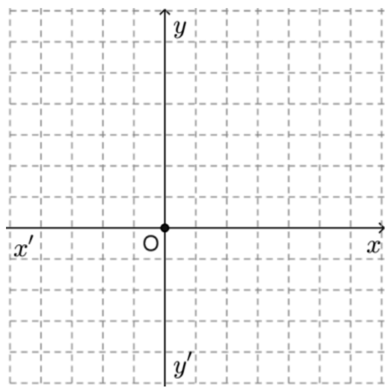
Table-2.

5) Join the plotted points in system of table-2.
6) What is the formed shape in each system? (straight line)
7) Does their prolongation (extention) pass through the origin? (yes)

Consider the third table:

Table-3				
	x	1	2	3
	y	3	5	7

a) Is the above table a proportionality table? Justify. ……………………………………….
b) Write an algebraic expression of y as a function of x: ……………………………………..
c) Graph the obtained expression:
d) Does its prolongation pass through the origin?
e) ……………………………………………….
f) Is the formed expression a linear function? Give two reasons.
   - ✓ ……………………………………….
   - ✓ ……………………………………….



Conclusions:

1) A linear equation is an expression whose graph is a straight line, so the expression $y = ax$ represents a linear equation
2) The graph of:
   - A linear function of equation: $y = ax$ is a straight line passing through origin.
   - An affine function of equation: $y = ax + b$ is a straight line that does not pass through origin (where a & b are non-zero real numbers)

Consider the equations the straight lines $(d_1): y=3x$, $(d_2): y=2x$, $(d_3): y=x$, $(d_4): y=-x$, $(d_5): y=-\frac{3}{2}x$ & $(d_6): y=-\frac{x}{2}$.

1) Sort with justification the above equations into two groups (linear and affine functions) :
   - Group-A: ……………………………………………
   - Group-B: ……………………………………………
2) Graph in the following orthonormal systems of axes straight lines of

Group-A:

………………

x		
y		
$(x;y)$		

………………

x		
y		
$(x;y)$		

………………

x		
y		
$(x;y)$		

Group-B:

………………

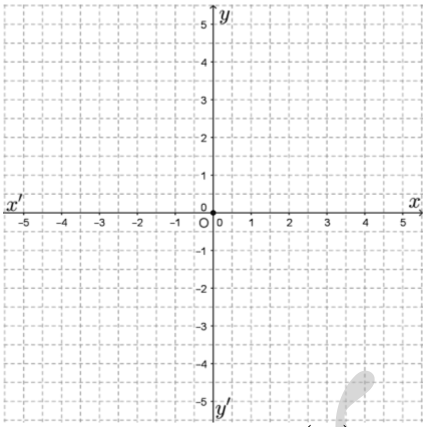
x		
y		
$(x;y)$		

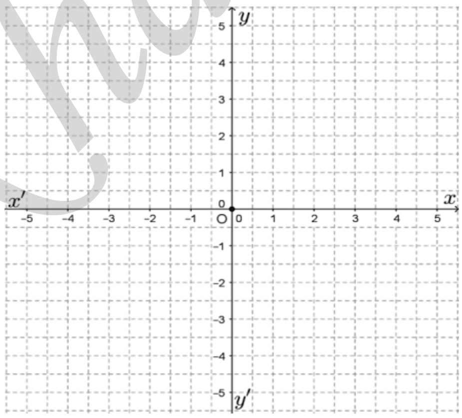
………………

x		
y		
$(x;y)$		

………………

x		
y		
$(x;y)$		

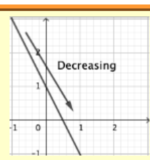




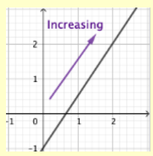
   a. Is the direction of (d_1), increasing or decreasing? ……………………………………
   b. What can you say about direction of straight lines with negative slope? (decreasing)
3) Compare slopes of:
   - Group-A: ……………………………………………
   - Group-B: ……………………………………………
4) Deduce the role of the slope (a) in the equation of the straight line? (Shows the direction of a line)

Conclusions:

- If $a>0$, then the straight line is increasing



If $a<0$, then the straight line is decreasing



5) Consider the straight lines $(l): y=3x-\sqrt{2}x$ & $(d): 2y-3x+1=0$
   a. Determine the slope (slant, director coefficient) of (l)& (d). (ans: $a_{(l)}=3-\sqrt{2}$ & $a_{(d)}=\frac{3}{2}$)
   b. Which straight line is steeper? Justify. ……………………………………………

Conclusions: As the ***slant*** of a straight-line ***increases***, then its graphical representation approaches y-axis