Lycée Des Arts | Mathematics | 9^{th}-Grade

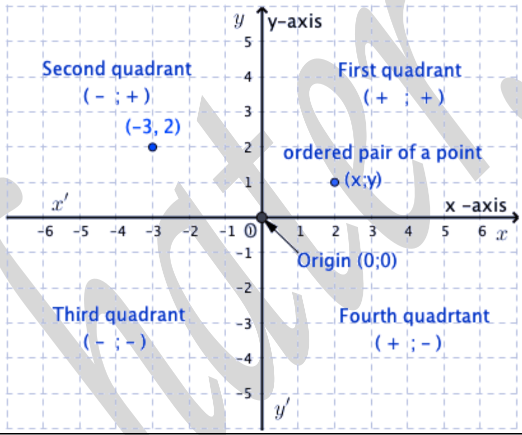
Name: . . . . | "Equation of a straight line" | A.S-7.1.

I- Locating a point on a system of axes:

Def: An orthonormal system of axes (x'Ox & y'Oy) is a system of two perpendicular axes with the same scale.

where: { The horizontal axis is called the ***abscissa axis*** *or* ***x − axis***
The vertical axis is called the ***ordinate axis*** *or* ***y − axis*** }

Note: The system of axes (x'Ox & y'Oy) cuts the plane into four parts each is called a quadrant.

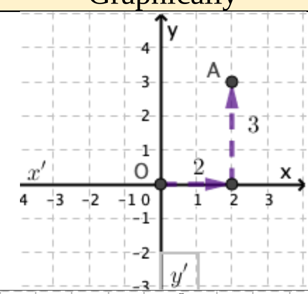
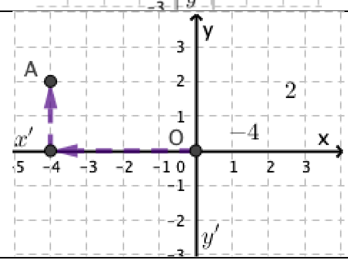


What do we need to locate a point on a plane?

To locate any point in a plane (x'Ox & y'Oy), we need ***two components***.	
The abscissa	The ordinate
is the 1^{st} component of the ordered pair $(x; y)$, it gives how far is a point from origin and in which sense is it moving along $x - axis$.	is the 2^{nd} component of the ordered pair $(x; y)$, it gives how far is a point from origin and in which sense is it moving along $y - axis$.

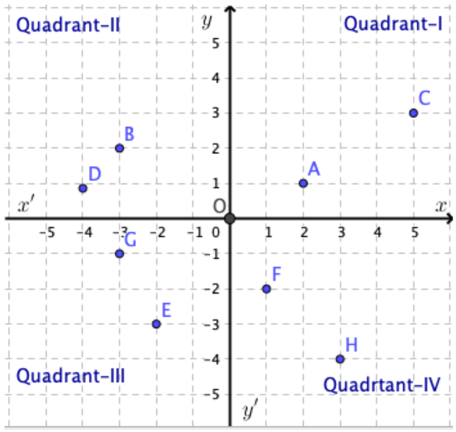
Point on an orthonormal system of axes $(x'Ox \,\&\, y'Oy)$	
Form	$A(x; y)$
Reading	A is a point of coordinates x & y
Notes	Value of x, is the abscissa or 1^{st} component Value of y, is the ordinate or 2^{nd} component

Application: Locate (Plot) on the system of axes$(x'Ox \,\&\, y'Oy)$ the points $A(2; 1)$ & $B(-4; 1)$

Point to locate	Explanation	Graphically
$A(2; 3)$ on $(x'Ox \,\&\, y'Oy)$	We start from the origin $O(0; 0)$ Since, $x_A = +2$ *and* $y_A = +3$, then 1^{st} move along $x - axis$ 2- steps in $+ve$ sense 1^{st} move along $y - axis$ 3- steps in $+ve$ sense	
$B(-4; 1)$ on $(x'Ox \,\&\, y'Oy)$	We start from the origin $O(0; 0)$ Since, $x_A = -4$ *and* $y_A = +1$, then 1^{st} move along $x - axis$ 4- steps in $-ve$ sense 1^{st} move along $y - axis$ 1- step in $+ve$ sense	

Ex$_1$: Consider in an orthonormal system of axes
$(x'Ox \;\&\; y'Oy)$the points $A, B, C, D\ , E, F \;\&\; G$

1) Determine graphically the coordinates of given points.
2) To which quadrant does each point belong?



Ex$_2$: Consider the system $(x'Ox \;\&\; y'Oy)$:

1) Locate $A(0;4), B(0;1), D(0;-2) \;\&\; H(0;-2.5)$
2) To which axis do the above points belong?
3) What do you notice about abscissas of above points?
4) What do you conclude?

Ex$_3$: The plane is considered as the orthonormal system $(x'Ox \;\&\; y'Oy)$:

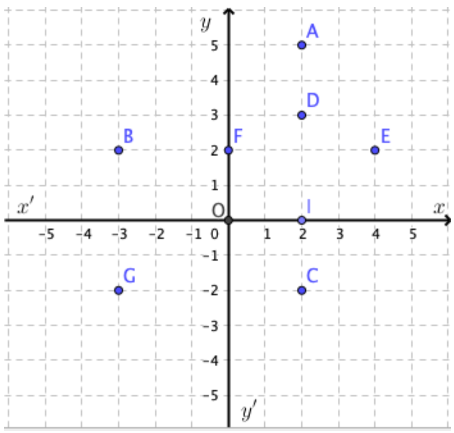
1) Locate $E(3;0), F(-2;0), J(1.5;0) \;\&\; S(-2.5;0)$
2) To which axis do the above points belong?
3) What do you notice about the abscissas of the above points?
4) What do you conclude?

Conclusion: Any point on the $\boldsymbol{x-axis}$, has a zero ordinate & it is of the form $(x;0)$.
Any point on the $\boldsymbol{y-axis}$, has a zero abscissa & it is of the form $(0;y)$.

II- *Lines parallel to either of the coordinate axes:*

Reminder: A straight line is a set of infinite number of collinear points in plane.

Ex$_1$: Consider the orthonormal system of axes $(x'Ox \;\&\; y'Oy)$:

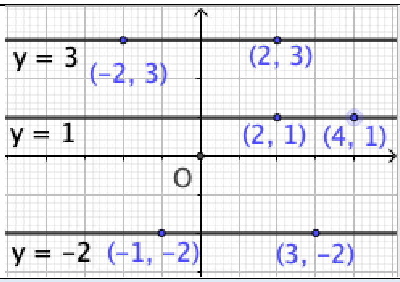
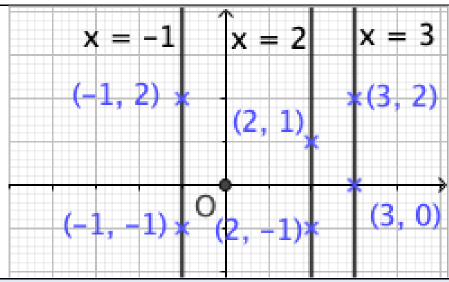


1) Detetmine the coordinates of the points $A, C \;\&\; D$.
   a. Compare the coordinates of the points $A \;\&\; C$?
   b. What is the relative position of line $(AC) \;\&\; y-axis$?
   c. What do you conclude?
2) Detetmine the coordinates of the points $B, F \;\&\; E$.
   a. Compare the coordinates of the points $B \;\&\; F$?
   b. What is the relative position of line $(BF) \;\&\; x-axis$?
   c. What do you conclude?
   d. Complete with best word. (same- different)

Any set of points on a line parallel to $x-axis$ have……… ordinate.

3) Prove that (BG) is parallel to $y'Oy$.
4) Deduce the relative position of $(AC) \;\&\; (BE)$.

Conclusions: 1) Any two points having same ordinate, the line joining them is parallel to $x-axis$.
2) Any two points having same abscissa, the line joining them is parallel to y$-axis$.

Graphical study		
Analytical study	A line parallel to $x-axis$ is of equation $y = cst$	A line parallel to y$-axis$ is of equation $x = cst$

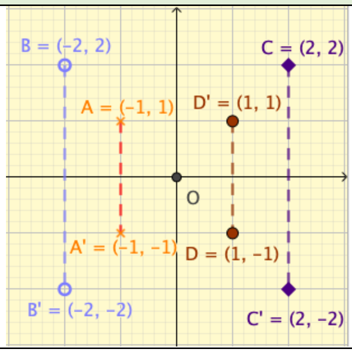
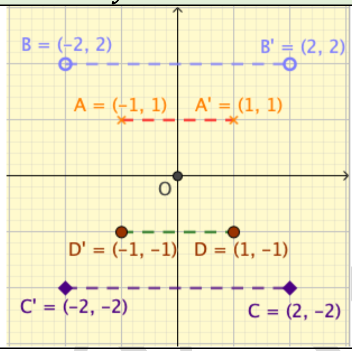
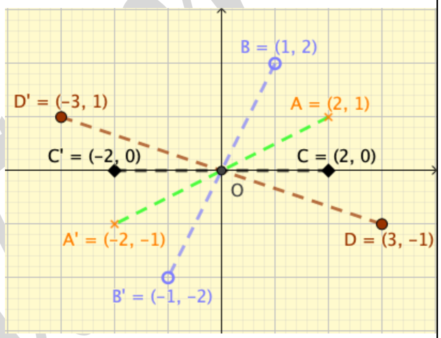
III- *How to find coordinates of a midpoint?*

> **Rule:** To find the coordinates of I, the midpoint of $[AB]$, *where* $A(x_A; y_A)$ & $B(x_B; y_B)$
> Use the formulas: $x_I = \frac{(x_A+x_B)}{2}$ and $y_I = \frac{y_A+y_B}{2}$

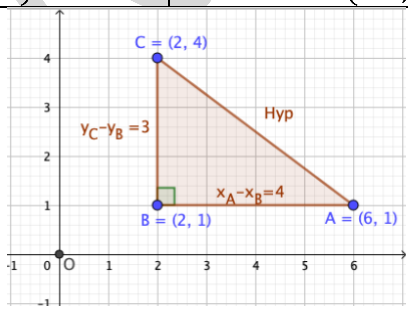
Ex: Consider in the orthonormal system $(x'Ox$ & $y'Oy)$ the points: $A(3;4)$& $B(-2;1)$

1. Find the coordinates of I, the midpoint of $[AB]$.
2. Determine the coordinates of S, the symmetric of A with respect to B.

IV- *Symmetry of a point:*

Symmetry w.r.t	$x - axis$	$y - axis$	Origin
Graphically			
What do you notice?	Points have ***same abscissas***, but ***opposite ordinates***	Points have ***same ordinates***, but ***opposite abscissas***	Points have ***opposite coordinates***
Coordinates	If $R(a;b)$ then its symmetry $R'(a;-b)$	If $S(a;b)$ then its symmetry $S'(-a;b)$	If $K(a;b)$ then its symmetry $K'(-a;-b)$

V- *How to find length of a segment?*



Consider in orthonormal plane the points

1) Compare:
   a. Abscissas of B & C
   b. Ordinates of A & B
2) To find length of AB, which is ∥ $x - axis$, we use: $AB = x_B - x_A = 6 - 2 = 4units$
3) To find length of BC, which is ∥ $y - axis$, we use: $BC = y_C - y_B = 4 - 1 = 3units$
4) If ABC is right triangle at B, then is it true that using Pythagorean theorem, we can write $AC^2 = (x_C - x_A)^2 + (y_C - y_A)^2$
5) Use the above rule to find AC.

> **Rule:** To find the distance between any two points $A(x_A; y_A)$ & $B(x_B; y_B)$, or the length of $[AB]$
> Use the formula:
> $$AB^2 = (x_B - x_A)^2 + (y_B - y_A)^2$$

Ex: Consider in the system $(x'Ox$ & $y'Oy)$ the points: $A(3;4)$ $B(2;1)$ & $C(5;0)$

1) Plot the given points
2) Find the length of the segments $[AB]$ *and* $[BC]$.
3) Determine the nature of triangle ABC.
4) Calculate the coordinates of I, the center of the circle circumscribed about ABC.
5) Find the coordinates of D, the symmetric of B with respect to I.
6) What is the nature of quadrilateral $ABCD$? Justify.

Applications

1) Choose with the appropriate ***justification*** the correct answer.

No.	*Statements*	*Proposed answers*		
		A	*B*	*C*
1.	The lines $x = 1$ & $x = -1$. are	Parallel	Perpendicular	Parallel to $x'ox$.
2.	The lines $x = 1$ & $y = -3$. are	Parallel	Perpendicular	Parallel to $x'ox$.
3.	The line $y = -1$ cuts the y-axis at:	1	-1	Does not have
4.	The line $y = -3$ passes through	$(0;-3)$	$(-3;0)$	$(0;0)$
6.	The lines $y = -3$ and $x = 2$ intersect at the point	$(-3;2)$	$(-2;3)$	$(2;-3)$

2) Consider the following cartesian systems:

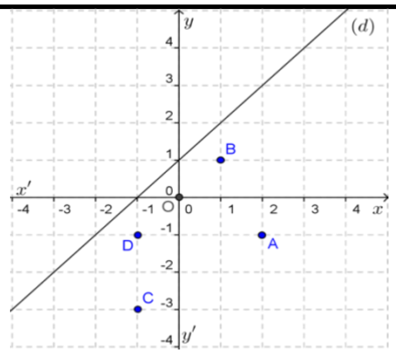


Fig-1.

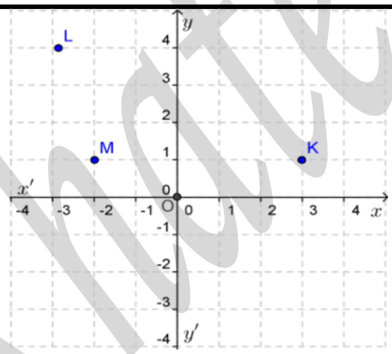


Fig-2.

Find & locate the coordinates of the points:

a- ***R*** the symmetric of ***A*** *w. r. t* origin.
b- ***N*** the symmetric of ***A*** *w. r. t* ***B***.
c- ***E*** the symmetric of ***C*** *w. r. t* x-axis.
d- ***Q*** the symmetric of ***C*** *w. r. t* y-axis.

Find & locate the coordinates of the points:

i- ***S*** the orthogonal projection of ***K*** on x-axis.
ii- ***T*** the orthogonal projection of ***L*** on y-axis.

3) If $R(3a-2;1)$ & $N(1;b^2-4)$ are any points in the plane, then determine the values of a & b, so that:

a. N belongs to the $x-axis$
b. R belongs to the $y-axis$

4) If $N(0;1)$ & $P(a;a-1)$ are any two points, then find a so that triangle NOP is isosceles $O(0;0)$.

5) Consider in an orthonormal system of axes, the points $A(1;3), B(5;5) \& N(5;3)$.

a) Plot the given points.
b) Compute the coordinates of I the midpoint of $[AB]$.
c) What is the nature of triangle ABN?
d) Deduce that points $A, B \& N$ belong to a circle (***C***), whose center is to be determined.

6) Consider in the coordinate system $x'Ox, y'Oy$ the points: $A(-1;0), B(1;-4) and\ C(-9;-4)$.

a. Plot the given points.
b. Determine the nature of the formed triangle.
c. Compute the area of triangle ABC.
d. Determine the coordinates of point I the center of gravity of the formed triangle.
e. Find the center and the radius of the circle circumscribed about triangle ABC.
f. Find the coordinates of point D the fourth vertex of the parallelogram $ABCD$.

7) Consider the points: $R(0;-1), P(4;-1) and\ N(2;5)$.

i. Prove that triangle *RPN* is isosceles of vertex *N*.
ii. Deduce the coordinates of *H*, the orthogonal projection of *N* on (*RP*).