Lycée Des Arts — Mathematics — 9^{th}-Grade

Name: . . . . — "Equation of a straight line" — A.S-7.4.

Reminder: A straight line is a set of infinite number of collinear points in plane.

I- *How to prove that three points in a plane are collinear?*

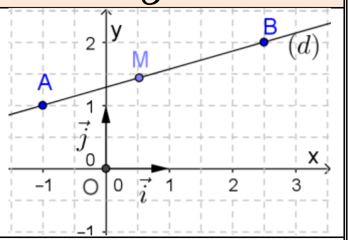
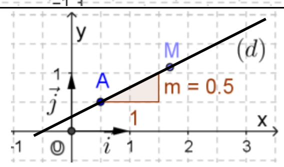
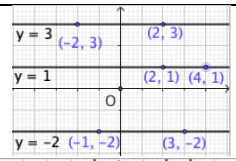
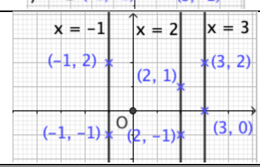
To prove that three points in a plane are collinear we have two main strategies:

Strategy	1	2
	If any of the two points are on a given line	If you are given any 3 points
Method	Just prove that the 3rd point is on the line, by replacing its coordinates in eqn of given line.	Just prove that the 3 points have equal slopes.
Application	Ques: Prove that A, B & C collinear, so that $(AB): y = 2x - 3$ & $C(2; 1)$	Ques: Prove that A, B & C collinear, so that $A(1; 3), B(4; 2)$ & $C(7; 1)$
	Soln: $(AB): y = 2x - 3$ & $C(2; 1)$ C is on (AB), if its coordinates fit in eqn of (d) $y_C = 2x_C - 3$ $1 \overset{?}{=} 2(2) - 3$ $1 = 1$ ✔ Thus, points A, B & C are collinear.	Soln: $a_{(AB)} = a_{(AC)}$ $\frac{y_B - y_A}{x_B - x_A} = \frac{y_C - y_A}{x_C - x_A}$ $\frac{2-3}{4-1} \overset{?}{=} \frac{1-3}{7-1}$ $\frac{-1}{3} = \frac{-2}{6}$ ✔ Thus, points A, B & C are collinear.

II- *How to determine the equation of a straight line in plane?*

At this level to determine the equation of a line we need		
The coordinates of two points $A(x_A; y_A)$ & $B(x_B; y_B)$	**Or**	Coordinates of a point and a slope $A(x_A; y_A)$ & a

➢ To find equation of a straight line it is ***sufficient*** to use ***one*** of the following ***combinations***:

Given information	*Formula used and steps to follow*	*Figures*
Line passing through two given points $A(x_A; y_A)$ & $B(x_B; y_B)$	✔ Take any point $M(x, y)$ on the given straight line. Now, points A, B & $M$are collinear, Then $a_{(AM)} = a_{(AB)}$ Thus, $\frac{y-y_A}{x-x_A} = \frac{y_B-y_A}{x_B-x_A}$	
Line passing through a given point $A(x_A; y_A)$ and knowing its slope (director coefficient)	✔ Take any point $M(x, y)$ on the given straight line. $\frac{y - y_A}{x - x_A} = a$ *OR* $y - y_A = a(x - x_A)$	
Line passing through two given points $A(x_A; h)$ & $B(x_B; h)$ having same ordinates.	✔ $A(x_A; h)$ & $B(x_B; h)$ have same ordinate so, $(AB) \parallel x - axis$ thus, $(AB): y = h$	
Line passing through two given points $A(v; y_A)$ & $B(v; y_B)$ having same abscissas.	✔ $A(x_A; h)$ & $B(x_B; h)$ have same abscissa so, $(AB) \parallel y - axis$ thus, $(AB): x = v$	

Exercises

1) Consider in the system $(x'Ox \; \& \; y'Oy)$ the points: $A(3;4)$ $B(2;1)$ & $C(5;0)$
   a. Plot the given points.
   b. Prove by calculation that given points are not collinear.

2) Find the numerical value of m, so that the given points $A(2;m)$ $B(1;1)$ & $C(m;5)$ are collinear.

3) Determine the equation of the straight line (d) in each of the following cases and **trace it**:
   a. Passing through the points: $A(-3;2)$ & $B(3;5)$.
   b. Passing through the points:$M(4;1)$ & $N(4;-2)$.
   c. Passing through the points:$E(2;1)$ &. $F\left(\frac{2}{3};1\right)$.
   d. Passing through the points:$S(2;3)$ & $K(4;6)$.

4) Determine the equation of the straight line(r) in each of the following cases:
   a. The parallel to $(l): y - 3x = \sqrt{2}x + 1$ and passing through the point $B(\frac{3}{2};-\frac{1}{2})$
   b. The perpendicular to $(n): 4x - 2y + 3 = 0$ & passing through point $E(5,-1)$.
   c. The perpendicular bisector of the segment [AB], where $A(1;-2)$ & $B(2;3)$.