Lycée Des Arts	*Mathematics*	*8^{th}-Grade*
Name:	*"Square roots"*	*A.S-8.3*

Reminder: Radicals are nothing but fractional powers so whatever works with powers basically works with radicals

Operations on radicals

A. Multiplication and division:

☆ How to express $\sqrt{b} \times \sqrt{d}$, where c & d are positive real numbers in form of one radical

1. Observe and write in form of one radical (**one power**):

One exponent	$2^3 \times 5^3 = (5)^3$	$11^7 \times 3^7 =$	$\frac{14^5}{9^5} = \left(\frac{14}{9}\right)^5$	$\frac{17^8}{21^8} =$
One radical	$\sqrt{7} \times \sqrt{3} = \sqrt{7 \times 3}$	$\sqrt{2} \times \sqrt{13} =$	$\frac{\sqrt{5}}{\sqrt{11}} = \sqrt{\frac{7}{11}}$	$\frac{\sqrt{15}}{\sqrt{13}} =$

☆ Ex-1. Observe and express in simplest form possible:

$\sqrt{3} \times \sqrt{3} = \left(\sqrt{3}\right)^2 = 3$ $\qquad$ $\sqrt{5} \times \sqrt{12} = \sqrt{5} \times \sqrt{2^2 \times 3} = 2\sqrt{15}$

$\sqrt{5} \times \sqrt{5} =$ $\qquad$ $\sqrt{7} \times \sqrt{18} =$ $\qquad$ $\sqrt{5} \times \sqrt{5} =$

$\sqrt{11} \times \sqrt{11} =$ $\qquad$ $\sqrt{13} \times \sqrt{50} =$ $\qquad$ $\sqrt{7} \times \sqrt{18} =$

☆ How to express $a\sqrt{b} \times b\sqrt{d}$ in form of $e\sqrt{f}$, where c, d & f are positive real numbers

To express $a\sqrt{b} \times b\sqrt{d}$, in form of $e\sqrt{f}$	
We multiply coefficient by coefficient and radicand by radicand	$3\sqrt{5} \times 2\sqrt{7} = (3 \times 2)\sqrt{5 \times 7} = 6\sqrt{35}$

☆ Ex-2. Observe and write in form of $e\sqrt{f}$, where f is a positive real number:

$2\sqrt{3} \times 5\sqrt{7} = (2 \times 5)\sqrt{3 \times 7} = 10\sqrt{21}$ $\qquad$ $3\sqrt{5} \times 2\sqrt{15} = 6\sqrt{5^2 \times 3} = 30\sqrt{3}$

$2\sqrt{11} \times 7\sqrt{3} =$ $\qquad$ $3\sqrt{6} \times 5\sqrt{8} =$

$-2\sqrt{5} \times 3\sqrt{2} =$ $\qquad$ $13\sqrt{6} \times \sqrt{8} =$

Conclusion: if $a \geq 0$ & $b > 0$ then we can write

Expression	Algebraically	In words
$\sqrt{a} \times \sqrt{a}$	a	Multiplying the radical by itself gets the radicand out
$\sqrt{a} \times \sqrt{b}$	$\sqrt{a \times b}$	To multiply two radicals, we multiply the radicands
$a\sqrt{b} \times b\sqrt{d}$	$(a \times b)\sqrt{b \times d}$	We multiply coefficient by coefficient and radicand by radicand

$\dfrac{\sqrt{a}}{\sqrt{b}}$	$\sqrt{\dfrac{a}{b}}$	To divide two radicals, we divide the radicands

B. *Addition and subtraction:*

Observe how we can express the following in simplest form possible:

Expression	*Detailed solution*	*Explanation*
$2x + 3(x+1)$	$\underbrace{2x} + \underbrace{3x} + 3 = 5x + 3$	Add coefficients of similar monomials
$2\sqrt{7} + 3(\sqrt{7} + 1)$	$2\sqrt{7} + 3\sqrt{7} + 3 = 5\sqrt{7} + 3$	Add coefficients of terms with same radicands

Application:

Ex-3.

Use a **calculator** to compute and many other similar examples of your choice

Expressions	Answer	Generalization
$A = 3\sqrt{2} + 5\sqrt{2}$		
$B = 2\sqrt{3} - 7\sqrt{3}$		
$C = -3\sqrt{5} + 11\sqrt{20}$		
$D = -5\sqrt{2} + 7\sqrt{2} + 3\sqrt{8}$		

Ex-4.

Express without using calculator the following in simplest form possible:

Expressions	Answer
$E = 5\sqrt{3} + 7\sqrt{3}$	
$F = -2\sqrt{5} - 3\sqrt{45}$	
$G = -3x\sqrt{2} + 11x\sqrt{8}$	
$H = 5\sqrt{2x-1} + 7\sqrt{18x-9} \quad x > 1$	

Conclusion: if $a \geq 0$, then we can write $b\sqrt{a} + c\sqrt{a} = (b+c)\sqrt{a}$

C. Rationalization:

- ✓ Def: Rationalization is to eliminate the radical sign
- ✓ To rationalize the denominator is to eliminate the radical from the denominator
- ✓ How to rationalize?
  - To rationalize we multiply both numerator and denominator of the fraction by the conjugate of the denominator

➯ Reminder: $(a+b)(a-b)=a^2-b^2$

➯ Find the term (factor), that if multiplied by the given term (factor) the radical will be eliminated:

No.	Term	Its conjugate	Product of the term by its conjugate	In general, Conjugate of
1	$\sqrt{2}$	$\sqrt{2}$	$\sqrt{2}\times\sqrt{2}=2$	$\sqrt{a}$ is $\sqrt{a}$
2	$3\sqrt{5}$	$\sqrt{5}$	$3\sqrt{5}\times\sqrt{5}=15$	$b\sqrt{a}$ is $\sqrt{a}$
3	$3\sqrt{2}-2\sqrt{3}$	$3\sqrt{2}+2\sqrt{3}$	$\left(3\sqrt{2}\right)^2-\left(2\sqrt{3}\right)^2=6$	$\left(b\sqrt{a}-c\sqrt{d}\right)$ is $\left(b\sqrt{a}+c\sqrt{d}\right)$
4	$2\sqrt{5}+1$	$2\sqrt{5}-1$	$\left(2\sqrt{5}\right)^2-(1)^2=19$	$\left(b\sqrt{a}+c\sqrt{d}\right)$ is $\left(b\sqrt{a}-c\sqrt{d}\right)$

➯ Application: Observe how we can eliminate the radical from the denominator of: (Rationalize the denominator)

- ✓ $\frac{3-\sqrt{2}}{\sqrt{5}}=\frac{3-\sqrt{2}}{\sqrt{5}}\times\frac{\sqrt{5}}{\sqrt{5}}=\frac{3\sqrt{5}-\sqrt{10}}{5}$
- ✓ $\frac{1+3\sqrt{5}}{\sqrt{5}-1}=\frac{\left(1+3\sqrt{5}\right)}{\left(\sqrt{5}-1\right)}\times\frac{\left(\sqrt{5}+1\right)}{\left(\sqrt{5}+1\right)}=\frac{1+\sqrt{5}+3\sqrt{5}+3\left(\sqrt{5}\right)^2}{\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)}=\frac{1+4\sqrt{5}+3(5)}{\left(\sqrt{5}\right)^2-(1)^2}=\frac{16+4\sqrt{5}}{4}=\frac{4\left(4+\sqrt{5}\right)}{4}=4+\sqrt{5}$

Ex-5. Rationalize the following:

1) $\frac{5-7\sqrt{6}}{3\sqrt{2}}$

2) $\frac{3-2\sqrt{5}}{2\sqrt{3}-1}$

3) $\frac{5-2\sqrt{3}}{2\sqrt{3}-3\sqrt{2}}$

Reminder:

Ex-6: Compute: $(3+5)^2 = \ldots\ldots\ldots$ $\qquad$ $(3^2-2^2)^2 = \ldots\ldots\ldots$

Ex-7. Calculate:

Values of a & b	$\sqrt{a^2+b^2}$	$\sqrt{a^2}+\sqrt{b^2}$	Compare: $\sqrt{a^2+b^2}$ & $\sqrt{a^2}+\sqrt{b^2}$
$a=1$ & $b=1$			
$a=5$ & $b=-4$			
$a=4$ & $b=5$			

a) What do you notice?

…………………………………………………………………………………

b) Is it true that: $\sqrt{a \pm b} \leq \sqrt{a}+\sqrt{b}$?

…………………………………………………………………

c) Complete the following table:

Compute the numerical value of	For $x=1$	For $x=0$	For $x=3$	For $x=5$
$A=\sqrt{(x-2)^2}$				

i- Is it true that $\sqrt{(x-2)^2} = x-2$, for all real values of x? Explain.

……………………………………………………………………………………………

ii- For what values of x, is $A=0$?

…………………………………………………………………

iii- Express $\sqrt{(x-2)^2}$ without radical sign. (indicate all cases)

………………………………………………………………………………

………………………………………………………………………………

………………………………………………………………………………

Remark: $$\sqrt{a^2} = \begin{cases} itself: +a & \text{if and only if } a > 0 \\ its\ opp: -a & \text{if and only if } a < 0 \\ null: 0 & \text{if and only if } a = 0 \end{cases}$$