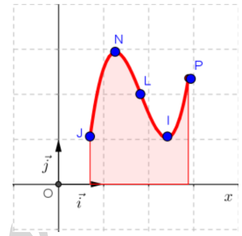
Al-Mahdi High School | Mathematics | 11^{th}-Grade
Name: . . . . | Function Derivative | A.S-8

Focusing events:

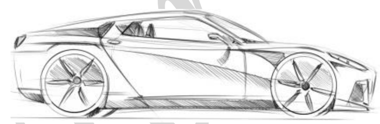
1- Consider the graph of a function f :



a) Between which two consecutive points is the average rate of change of f the greatest?

b) At what points is the instantaneous rate of change of f is positive, negative and zero?

2- A car is parked with its windows & doors closed for five hours. The temperature inside the car is given by the function f defined by $f(t)=2\sqrt{t^3}+17$, where t is time for which the car is first closed.



a) Find the average rate of change of the temperature from $t=1hr$ to $t=4hrs$.

b) Find the function that gives the instantaneous rate of change of the temperature for any time t, where $0<t<5$.

Geometric meaning of derivatives:

I- Consider in the system of axes $(O,\vec{i},\vec{j})$ the functions f & g defined respectively by: $f(x)=x^2-1$ & $x \longrightarrow 2x-1$ & their representative curves C_f & (d) over $\mathbb{R}$.

a. Draw on same graph the straight line: (d).

x		
y		
(x,y)		

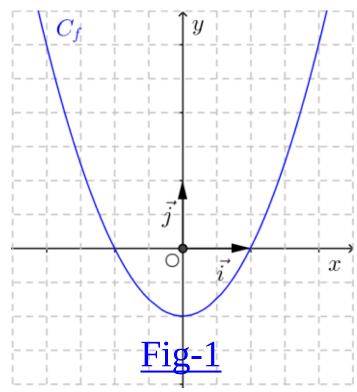


Fig-1

b. Find graphically the coordinates of $A(x_A, y_A)$ and $B(0, y_B)$ the points of intersection of C_f & (d). …………………………

c. What does the equation (E) defined by $f(x)=g(x)$ tell you?

……………………………………………………………………

d. Solve (E) ……………………………………………………...

e. Answer by ***true*** or ***false***, and justify ***each***:

i. (d) is tangent to C_f. ……………………………………………

ii. (d) is secant to C_f …………………………………………..........

f. What happens to (d) as the point: 1) A approaches B? ………………………………..

2) B approaches A? ………………………………..

g. How many tangents, does C_f admit? …………………………………………………

II- Let f & g be two functions in the system $(O,\vec{i},\vec{j})$, where f is defined by: $f(x)=x^2$ and the representative curves C_f & (d) over $\mathbb{R}$.

a. Use the adjacent graph to determine the equation of (d).

……………………………………………………………………...

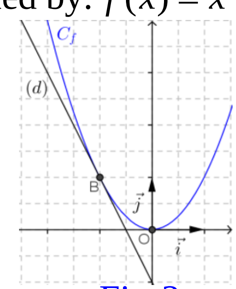


Fig-2

b. Prove that the equation $(E): g(x)=f(x)$ admits one double root, to be determined.…………………………………………………....

c. What does (d) represent to C_f ? ....

d. Indicate the slope of (d) ....

e. Calculate $r=\lim\limits_{x\to x_B}\dfrac{f(x)-f(x_B)}{x-x_B}$ ....

f. Compare the value of r with the slant of (d) ....

g. What does r represent? ....

Def:	The derivative of a function at a point of abscissa x_o is denoted by $f'(x_o)$ where $f'(x_o)$ is the slope of the tangent (T) to C_f at x_o. $f'(x_o)=\lim\limits_{x\to x_o}\dfrac{f(x)-f(x_o)}{x-x_o}$

- Reminder: How to find equation of a straight line:

Passing through any two given points $A\ \&\ B$	$\dfrac{y-y_A}{x-x_A}=\dfrac{y_B-y_A}{x_B-x_A}$
Having a slope a and a point A	$y-y_A=a(x-x_A)$

III- Find the equation of $A\ \&\ B$ the tangent line to the curve of the function $f:f(x)=x^2-1$ at a point A of abscissa $x=1$.

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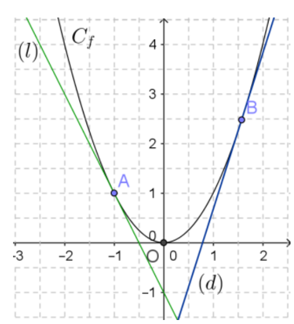
- Outcomes: - If $f'(a)=k$ and $k\neq 0$, then the C_f admits a unique tangent of the form $y=kx+b$
  - If $f'(a)=k$ and $k=0$, then the C_f admits a horizontal tangent of the form $y=b$
  - If $f'(a)=\pm\infty$, then the C_f admits a .... tangent of the form ....

IV- In the adjacent graph, as $A\ \&\ B$ varies on C_f, the straight lines $(l)\&(d)$ remain tangent to C_f at $A\ \&\ B$ for all $x\in]-\infty,0[\ \&\ x\in]0,+\infty[$ respectively.



a. Complete the table:

Values of x	$]-\infty,0[$	$]0,+\infty[$
Sign of slope of $(l)\&(d)$		
Sense of variation of C_f		

b. What do you notice? ....

c. Complete the following statements:

- If $f'(a)>0$, then the C_f is ....at the point of abscissa $x=a$.
- If $f'(a)<0$, then the C_f is ....at the point of abscissa $x=a$.
- If $f'(a)=0$, then the C_f is ....at the point of abscissa $x=a$.