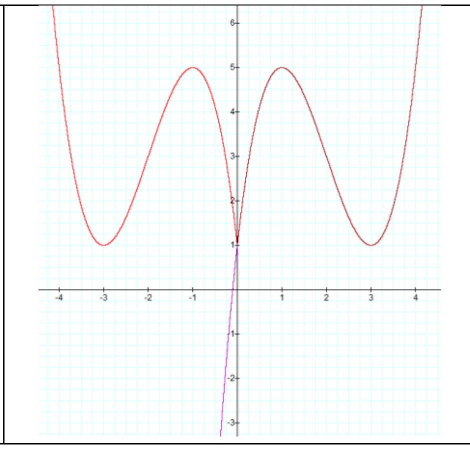
The Islamic Institution For Education & Teaching Al-Mahdi Schools	Answer Key	Mathematics Department
Class: Grade 11 (Scientific)	**Mid Year Exam**	**2014-2015**

Q	Part		Elements of the Answer	Mark
I	1	a	$g'(x) = \frac{-8x}{(x-1)^3}$	1
		b	$h'(x) = \frac{-1}{(2x-3)\sqrt{2x-3}}$	1
	2	a	$-1 \le \cos x \le 1$; $-3 \le 3\cos x \le 3$; $-4 \le 3\cos x - 1 \le 2$; but $x^2 + 2 > 0$, $\frac{-4}{x^2+2} \le f(x) \le \frac{2}{x^2+2}$	1
		b	$\frac{-4}{x^2+2} \le f(x) \le \frac{2}{x^2+2}$; $\lim\limits_{x\to+\infty}\left(\frac{-4}{x^2+2}\right) \le \lim\limits_{x\to+\infty} f(x) \le \lim\limits_{x\to+\infty}\left(\frac{2}{x^2+2}\right)$; $0 \le \lim\limits_{x\to+\infty} f(x) \le 0$; $\lim\limits_{x\to+\infty} f(x) = 0$	0.5
	3	a	$\lim\limits_{x\to4^-} f(x) = \lim\limits_{x\to4^+}\frac{x^2-5x+4}{18x-72} = \lim\limits_{x\to4^+}\frac{(x-1)(x-4)}{18(x-4)} = \lim\limits_{x\to4^+}\frac{x-1}{18} = \frac{1}{6}$; $\lim\limits_{x\to4^+} f(x) = \lim\limits_{x\to4^+}\frac{3-\sqrt{x+5}}{4-x} = \lim\limits_{x\to4^+}\frac{3-\sqrt{x+5}}{4-x}\times\frac{3+\sqrt{x+5}}{3+\sqrt{x+5}} = \lim\limits_{x\to4^+}\frac{4-x}{(4-x)(3+\sqrt{x+5})}$ $= \lim\limits_{x\to4^+}\frac{1}{3+\sqrt{x+5}} = \frac{1}{6}$	1 1
		b	$\lim\limits_{x\to4^-} f(x) = \lim\limits_{x\to4^+} f(x) = f(4)$; $b - 1 = \frac{1}{6}$; $b = \frac{7}{6}$	0.5
II	1		$m^2 + 6m + 5 = 0$; $m = -1$ or $m = -5$	0.5
	2		$\Delta > 0$; $-8m - 4 > 0$; $m < -\frac{1}{2}$	1
	3		$\frac{mS}{P} \ge 0$; $\frac{2m(m-1)}{m^2+2} \ge 0$; $m < 0$ or $m > 1$. But the roots exist when $m \le -\frac{1}{2}$. Thus $m \le -\frac{1}{2}$	1
III	1		$U_1 = 2$ and $U_2 = \frac{5}{2}$	0.25 0.25
	2		$U_2 - U_1 \ne U_1 - U_0$ since $U_2 - U_1 = \frac{1}{2}$ and $U_1 - U_0 = 1$; then (U_n) is not arithmetic $\frac{U_2}{U_1} \ne \frac{U_1}{U_0}$ since $\frac{U_2}{U_1} = \frac{5}{4}$ and $\frac{U_1}{U_0} = 2$; then (U_n) is not geometric	0.5 0.5
	3		$U_{n+1} - U_n = \frac{1}{2^n}$; (U_n) is strictly increasing since $U_{n+1} - U_n > 0$	0.25 0.25
	4	a	$\frac{V_{n+1}}{V_n} = \frac{1}{2}$ which is a constant, then (V_n) is a geometric sequence; $r = \frac{1}{2}$ and $V_0 = 1$	0.5 0.25 0.25
		b	$V_n = \frac{1}{2^n}$; $V_{10} = \frac{1}{2^{10}} = \frac{1}{1024}$	0.25 0.25
		c	$S = \frac{V_0(1-r^{n+1})}{1-r} = 2 - \frac{1}{2^n}$	0.5

IV	**1**		$(x-2)^2+(y-3)^2=13$; I(2 , 3) and $R=\sqrt{13}$ u	**0.5**
	2		$BI=\sqrt{26}$ u; BI > R, then B is at the exterior of the circle	**0.25** **0.25**
	3	**a**	$m(1)-(-2)-m-2=0$	**0.5**
		b	(d_m) tangent to (C); $d(I,(d_m))=R$; $\frac{\lvert m(2)-3-m-2\rvert}{\sqrt{m^2+1}}=\sqrt{13}$; $\lvert m-5\rvert^2=13(m^2+1)$; $6m^2+5m-6=0$; $m=\frac{3}{2}$ or $m=\frac{2}{3}$	**1**
		c	The tangent lines are of equations: $y=\frac{3}{2}x-\frac{7}{2}$ and $y=\frac{2}{3}x-\frac{8}{3}$	**0.5**
V	**1**		$\cos\left(\frac{\pi}{12}\right)\sin\left(\frac{5\pi}{12}\right)+\sin\left(\frac{\pi}{12}\right)\cos\left(\frac{5\pi}{12}\right)=\sin\left(\frac{\pi}{12}+\frac{5\pi}{12}\right)=\sin\left(\frac{\pi}{2}\right)=1.$	**1**
	2		$\tan(2x)-\tan(x)=\frac{\sin(2x)}{\cos(2x)}-\frac{\sin(x)}{\cos(x)}=\frac{\sin(2x)\cos(x)-\sin(x)\cos(2x)}{\cos(2x)\cos(x)}=\frac{\sin(2x-x)}{\cos(2x)\cos(x)}=\frac{\tan(x)}{\cos(2x)}$	**1.5**
	3		$\cos(2x)=1-2\sin^2(x)=\frac{1+\sqrt{5}}{4}$ $\cos(4x)=2\cos^2(x)-1=\frac{-1+\sqrt{5}}{4}=\sin x$	**0.75** **0.75**
	4		$\left(\overrightarrow{AB},\overrightarrow{BC}\right)=\frac{\pi}{2}(2\pi)$; $\left(\overrightarrow{AD},\overrightarrow{CB}\right)=\pi(2\pi)$; $\left(\overrightarrow{OA},\overrightarrow{BC}\right)=\left(\overrightarrow{OA},\overrightarrow{AD}\right)=\left(-\overrightarrow{AO},\overrightarrow{AD}\right)=\pi+\left(\overrightarrow{AO},\overrightarrow{AD}\right)=\pi+\frac{\pi}{4}=\frac{5\pi}{4}(2\pi).$	**0.25** **0.25** **0.5**
VI	**1**		$f'(x)=\frac{-ax^2-2bx+6x+a}{(x^2+1)^2}$	**0.75**
	2		$A\in(C)$; $b=3$ (AB) tangent to (C); $f'(x_A)=$ slope of (AB) = 2; $f'(0)=2$; $a=2$	**0.25** **1**
VII	**1**		$\lim_{x\to-\infty}f(x)=-\infty$ and $\lim_{x\to+\infty}f(x)=+\infty$	**0.25** **0.25**
	2		x: $-\infty$ … 1 … 3 … $+\infty$ f′(x): + 0 − 0 + f(x): $-\infty$ ↗ 5 ↘ 1 ↗ $+\infty$	**1**
	3		On the interval $]-\infty,1[$: f is continuous and strictly increasing, and f(x) changes signs from $-\infty$ to +1; then the equation f(x) = 0 has a unique root α. f(–0.2) = and f(–0.1) = ; f(–0.2) ×f(–0.1) < 0, then –0.2 < α < –0.1	**0.5** **0.5**
	4		$y=9x+1$	**1**
	5		At x = 2: $f''(2)=0$ and $f''(x)$ changes signs (– to +). I(2 , 3) x: $-\infty$ … 2 … $+\infty$ f″(x): − 0 +	**0.75** **0.25**
	6 (1 pt)			
	7		f(x) > 0 when x > α.	**0.5**
	8	**a**	$D_g=\mathbb{R}$ centered at zero and $g(-x)=f(\lvert -x\rvert)=f(\lvert x\rvert)=g(x)$	**0.5**
		b	If x ≥ 0: (G) ≡ (C) If x < 0: (G) is the symmetric of the first drawn part with respect to the ordinate axis	**0.5** (cnstrct) **0.5** (G)