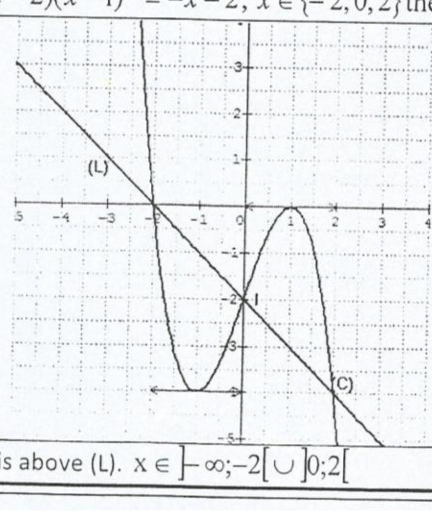
The Islamic Institution for Education & Teaching Al-Mahdi Schools	IN HIS NAME	Mathematics Department
		Scholastic Year :2015- 2016
		Date:
Grade 11(Sc)		Duration: 150 min
Name:	Mid-Year Exam	Mark: 40 points

Questions		Answers	Note
I-(5pts)	1)	$P = -m^2 - 5 = -(m^2+5) \le 0$ for $x \in \mathbb{R}$ **(C)**	1
	2)	Let $t = x^3$, $t^2 - 5t + 4 = 0$, $t_1 = 1$ and $t_2 = 4$ then $x_1 = 1$ and $x_2 = \sqrt[3]{4}$ **(A)**	1
	3)	$\lim\limits_{x\to1}\frac{x^2+x-2}{2x^2-3x+1} = \lim\limits_{x\to1}\frac{(x-1)(x+2)}{(x-1)(2x-1)} = \lim\limits_{x\to1}\frac{x+2}{2x-1} = 3$ **(C)**	1
	4)	$f(1)\times f(2) < 0$ then $(3m-2)(8m+1) \le 0 \ldots m \in \left]-\frac{1}{8};\frac{2}{3}\right[$ **(B)**	1
	5)	$f'(x) = 2\sin(x^2)2x\cos(x^2) = 2x\sin(2x^2)$ **(A)**	1
II-(3pts)	1.a)	$\Delta \ge 0$, $m^2+5m+4 \ge 0$, $m \in]-\infty; -4] \cup [-1; +\infty[$	1
	1.b)	$S = -2(m+2)$ & $P = -m$, $2S - P + 8 = m^2$ then $m(m+3) = 0$, $m = 0$ (accepted) & $m = -3$ (rejected)	1
	2)	f is defined over $\mathbb{R}$ ... $x^2 + 2(m+2)x - m > 0$ for $x \in \mathbb{R}$, then $\Delta \le 0$......$m \in [-4;-1]$	1
III-(3pts)	1)	• $-1 \le -\cos x \le 1$ then $1 \le 2 - \cos x \le 3$ then $x+1 \le x+2-\cos x \le x+3$ • $x+1 \le f(x)$ then $\lim\limits_{x\to+\infty} f(x) \ge \lim\limits_{x\to+\infty}(x+1) = +\infty$ *then* $\lim\limits_{x\to+\infty} f(x) = +\infty$	1 1
	2)	$\lim\limits_{x\to0^-} f(x) = f(0) = \lim\limits_{x\to0^+} f(x) = 1$ then f is continuous at 0	1
IV-(5pts)	1)	$U_1 = \frac{8}{3}$ & $U_2 = \frac{28}{9}$	¼¼
	2)	$U_1 - U_0 \ne U_2 - U_1$ & $\frac{U_1}{U_0} \ne \frac{U_2}{U_1}$ then (U_n) is neither arithmetic, nor geometric	1
	3.a)	$\frac{V_{n+1}}{V_n} = \frac{U_{n+1}-3}{Un-3} = \frac{\frac{-1}{3}Un+4-3}{Un-3} = -\frac{1}{3}$, then (V_n) is a geometric sequence with common ratio $r = -\frac{1}{3}$ and the first term $V_0 = U_0 - 3 = 1$	1½
	3.b)	$V_n = V_0 \times r^n = \left(-\frac{1}{3}\right)^n$ $U_n = V_n + 3 = \left(-\frac{1}{3}\right)^n + 3$	1
	3.c)	$S_n = V_0 \times \frac{1-\left(-\frac{1}{3}\right)^{n+1}}{1+\frac{1}{3}} = \frac{3}{4}\left(1-\left(-\frac{1}{3}\right)^{n+1}\right)$ $T_n = S + 3(n+1) = \frac{3}{4}\left(1-\left(-\frac{1}{3}\right)^{n+1}\right) + 3(n+1)$	1

V-(4pts)	1)	I (2 ; −1) & R = $\sqrt{5}$	1
	2.a)	$x_A^2+y_A^2-4x_A+2y_A=0$ or $IA=\sqrt{5}$, then $A\in(C)$	½
	2.b)	(T) tangent to (C) at A, then (T) ⊥ (IA) Let M(x, y) ∈ (T) then $\overrightarrow{AM}.\overrightarrow{IA}=0$, $(x-3)+2(y-1)=0$, (T): $x+2y-5=0$	1
	3)	(d) tangent to (C) then d (I, (d)) = $\sqrt{5}$ … $\frac{\|2+1+b\|}{\sqrt{2}}=\sqrt{5}$ .. $b=-3+\sqrt{10}$ or $b=-3-\sqrt{10}$	1½
VI-(6pts)	1)	f admits at x = −3 a local maximum, then f′(−3) = 0. f′(0) = slope of tangent to (C) at D = $\frac{2}{0.5}=4$ f′(2) = slope of tangent to (C) at E = slope of the straight line of eq. (y = − x) = −1	½ ½ 1
	2)	$-2\in D_f$ & (C) admits two semi-tangents at (−2; 0), then f is not differentiable at x = −2. The curve is not cut at (−2; 0) then f is continuous at (−2; 0)	½ ½
	3)	(T) // (d), then slope of (T) = −1 and E(2 ; 2) ∈ (T) then (T): y = − x + 4	1
	4.a)	f(x) ≠ 0 Then $D_h=]-\infty;-4[\cup]-4;-2[\cup]-2;1[\cup]1;+\infty[$	1
	4.b)	$h'(0)=\frac{-f'(0)}{f^2(0)}=\frac{-4}{4}=-1$	1
VII-(9pts)	A)	$f(2)=-4$ and $f'(2)=-9$ …. $\begin{cases}4a+b=-1\\12a+b=-9\end{cases}$ …. a = −1 & b = 3	1½
	B) 1)	$\lim\limits_{x\to-\infty}f(x)=+\infty$; $\lim\limits_{x\to+\infty}f(x)=-\infty$	½
	B) 2.a)	$f'(x)=-3x^2+3$. x: -∞ , −1 , 1 , +∞ f′(x): − 0 + 0 − f(x): +∞ ↘ −4 ↗ 0 ↘ −∞	1½
	B) 2.b)	$(-x-2)(x-1)^2=-x^3+3x-2=f(x)$	½
	B) 2.c)	f(0) = −2 …..the intersection with y-axis : (0, −2) f(x) = 0 ..$(-x-2)(x-1)^2=0$ ...x = −2 or x = 1 .. the intersection with x-axis: (−2, 0) and (1; 0)	¾
	B) 3)	$f(2x_1-x)+f(x)=f(-x)+f(x)=-4=2y_1$ then I is the center of symmetric of (C).	1
	B) 4)	$f(x)=-x+2$, $(-x-2)(x-1)^2=-x-2$, $x\in\{-2,0,2\}$ then (−2 ; 0), (0 ; −2), (2 ; −4)	1
	B) 5)		1½
	B) 6)	$f(x)>-x-2$ .. (C) is above (L). $x\in]-\infty;-2[\cup]0;2[$	¾

VIII- (5pts)	1)	$\cos\left(\frac{2\pi}{5}\right) = 2\cos^2\left(\frac{\pi}{5}\right) - 1 = \frac{-1+\sqrt{5}}{4}$	1½
	2.a)	$\frac{1-\cos(2x)}{\sin(2x)} = \frac{1-(1-2\sin^2 x)}{2\sin x\cos x} = \frac{2\sin^2 x}{2\sin x\cos x} = \tan x$ since that $\sin x \neq 0$.	1½
	2.b)	$\tan\left(\frac{\pi}{12}\right) = \frac{1-\cos\left(\frac{\pi}{6}\right)}{\sin\left(\frac{\pi}{6}\right)} = \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = 2-\sqrt{3}$	½
	3)	$\sin\left(\frac{\pi}{3}+x\right)\times\sin\left(\frac{\pi}{3}-x\right) = \left(\sin\left(\frac{\pi}{3}\right)\cos x + \cos\left(\frac{\pi}{3}\right)\sin x\right)\left(\sin\left(\frac{\pi}{3}\right)\cos x - \cos\left(\frac{\pi}{3}\right)\sin x\right)$ $= \left(\frac{\sqrt{3}}{2}\cos x\right)^2 - \left(\frac{1}{2}\sin x\right)^2 = \frac{3}{4}\cos^2 x - \frac{1}{4}\sin^2 x = \frac{3}{4} - \sin^2 x$	1½